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Application of Multiple Correlations Analysis in Portfolio Selection

Ruili Sun, Junpeng Jia, Shiguo Huang*

College of Mathematics and Information Science, Zhengzhou University of Light Industry, Zhengzhou 450001, China

**Author to whom correspondence should be addressed.*

Abstract: Portfolio selection based on the global minimum variance (GMV) model remains a significant focus in financial research. The covariance matrix, central to the GMV model, determines portfolio weights, and its accurate estimation is key to effective strategies. Based on the decomposition form of the covariance matrix. This paper introduces semi-variance for improved financial asymmetric risk measurement; addresses asymmetry in financial asset correlations using distance, asymmetric, and Chatterjee correlations to refine covariance matrices; and proposes three new covariance matrix models to enhance risk assessment and portfolio selection strategies. Testing with data from 30 stocks across various sectors of the Chinese market confirms the strong performance of the proposed strategies.

Keywords: Portfolio selection; GMV model; Semi-variance; Asymmetric correlation; Chatterjee correlation

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1. Introduction

The GMV model is a portfolio selection framework designed to achieve the lowest variance through optimized asset allocation. Unlike the classical mean-variance model, GMV focuses solely on minimizing risk rather than prioritizing future profitability [1]. This makes it widely applicable to portfolio selection problems, as seen in studies by many researchers [2, 3]. In the GMV model, the optimal portfolio weights depend exclusively on the covariance matrix of risky assets. The covariance matrix can be decomposed into variance and correlation, represented as the product of a diagonal variance matrix and a correlation coefficient matrix. Variance quantifies the risk of individual assets, while the correlation coefficient captures relationships between assets, facilitating effective risk diversification. Nonetheless, both variance and correlation coefficients have limitations in portfolio selection [4].

In assessing portfolio risk, researchers have found that semi-variance offers a more accurate measure of asymmetric risk. According to Mao, traditional methods treat positive and negative deviations equally, which does not align with actual investor psychology [5]. To address this, Artzner *et al.* emphasized the importance of focusing on adverse changes in returns, termed downside risk [6]. Wu *et al.* investigated dynamic mean-downside risk portfolio optimization under random interest rate changes in a continuous-time framework [7]. Rutkowska-Ziarko and Kliber used semi-variance to quantify downside risk, capturing investors' risk preferences more accurately [8]. Their findings revealed that investors exhibit decreasing risk aversion when ranking assets, showing a readiness to accept higher risks for greater rewards.

Asymmetric correlation plays a crucial role in portfolio selection, garnering significant attention in asset correlation analysis [9]. Longin and Solnik revealed that extreme market conditions increase correlations across international stock markets [10]. Ang and Chen found that correlations between assets intensify during market downturns, highlighting essential risk characteristics of investment portfolios [11]. Hong *et al.* introduced a model-free testing approach for analyzing asymmetric correlations in size and momentum portfolios, offering empirical support [12]. Meanwhile, Chuang *et al.* developed a nonparametric method to explore asymmetric co-movement in financial markets, finding significant evidence of such behavior in stocks and market indices [13]. These studies provide valuable theoretical insights, enabling investors to better understand and manage portfolio risk.

As researchers delve deeper into asset correlations, they have identified non-linear characteristics among financial assets. Distance correlation, introduced by Székely *et al.*, effectively captures these non-linear relationships, outperforming traditional Pearson correlation in risk characterization [14]. Extensions like Zhou's auto-distance correlation function (ADCF) and Székely *et al.* high-dimensional independence testing have enhanced its applicability [15, 16]. Meanwhile, Dueck *et al.* studied the problem of computing the distance correlation coefficient between Lancaster distribution class random vectors and derived a general series representation of the distance covariance of these distributions, enriching the theoretical foundation of distance correlation [17]. Applications in portfolio risk-return measurement further demonstrate its versatility and theoretical significance [18].

In this paper, three portfolio selection strategies within the GMV model is proposed, based on the variance-correlation decomposition of the covariance matrix while addressing asymmetry in both time and individual dimensions of financial risk assets. The key steps include: (i) replacing variance with semi-variance to account for investors' focus on downside risk; (ii) measuring correlations using distance, asymmetric, and Chatterjee correlations to capture time and individual asymmetries; (iii) constructing three novel covariance matrices and corresponding portfolio selection strategies to enhance effectiveness.

2. Preliminary works

In this section, some of the key preparatory works will be discussed in five aspects: the global minimum variance model, semi-variance, asymmetric correlation, distance correlation, and Chatterjee correlation.

2.1. Global minimum variance model

Given p risky assets, let $R_t = (r_{1t}, r_{2t}, ..., r_{pt})$ denote the p×1 vector of asset returns at time t where $t(t=1,2,...,n)$ denotes time, n is the sample size, and (·)T indicates the transpose operation. The traditional GMV model is expressed as,

$$min_{\omega}\ \omega^{\top}\Sigma\omega, \quad s.t. \quad \omega^{\top}1 = 1 \tag{1}$$

where ω is a p×1 weight vector, $\sum = Cov(R_t)$ is a p×p covariance matrix, and 1 is a p×1 vector of ones. The optimal portfolio weight for Equation (1) can be obtained as follows:

$$\omega^{\min}\left(\Sigma^{-1}\right)=\frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}}.$$

But the true covariance matrix Σ is unknown, and the sample covariance matrix $\widehat{\Sigma}$ is used to replace Σ , that is

$$\widehat{\omega}^{min}(\widehat{\Sigma}^{-1})=\frac{\widehat{\Sigma}^{-1}1}{1^{T}\widehat{\Sigma}^{-1}1} \tag{2}$$

Where $\widehat{\Sigma}=\frac{1}{n}\Sigma_{t=1}^{n}\left(R_t-\widehat{\mu}\right)\left(R_t-\widehat{\mu}\right)^{\top}$ and $\widehat{\mu}=\frac{1}{n}\Sigma_{t=1}^{n}R_t$. .

The covariance matrix can be decomposed into a variance-correlation form, which is the product of the diagonal matrix of variance and the correlation coefficients matrix. Variance measures risk and correlation coefficients measures the correlation of different financial assets. However, there are some shortcomings in using the traditional GMV model to deal with portfolio selection problems:

(1) Variance is a traditional risk measurement method. Although it has been widely used in many portfolio selection problems, it also has some practical limitations, such as considering any value above the mean as asset risk. Then semi-variance is presented to measure variance of portfolio reasonably.

(2) The eigenvalues of the sample covariance matrix do not match the eigenvalues of the true covariance matrix. The estimation error in the sample covariance matrix remains unresolved, and this error can lead to ineffective decisions in portfolio optimization and risk management. In the following sections, semi-variance, asymmetric correlation, distance correlation, and Chaterjee's correlation will be introducedone by one.

2.2. Semi-variance

Semi-variance does have significant value as a tool for assessing a portfolio's potential downside risk. Markovitz proposed and extended the application of semi-variance, respectively [1]. In recent years, many scholars have further studied and applied semi-variance. Huang assumes that ζ is a fuzzy variable with a finite expected value e, then the semi-variance S(ζ) of ζ is defined as follows [19]:

$$S[\zeta]=\ E\{[(\zeta-e)^{-}]^{2}\},$$

Where

$$(\zeta-e)^{-}=\begin{cases}\zeta-e & if\ \ \zeta\le e,\\ 0 & if\ \ \zeta>e.\end{cases}$$

Considering the above definition of semi-variance, the sample semi-variance can be further defined as:

$$\widehat{SVAR}_i=\frac{1}{n}\sum_{t=1}^{n}\left\{\left[\min\left(0,r_{it}-B\right)\right]^{2}\right\}$$

where $\widehat{SVAR}_i$ is the *i*-th value in the sample data set, r_{it} is the return value, *B* is the expected value of the sample data set, *n* is the number of non-zero terms.

2.3. Asymmetric correlation

By considering asymmetric correlation, investors, risk managers, and policymakers can better assess and manage market risks, develop more effective investment strategies, and create regulatory policies that contribute to greater stability and security across various market environments.

Suppose the two risky assets of return in period t are $\{r_{1t},r_{2t}\}$. Consider the extreme case of the two variables exceeding the correlation (i.e. exceeding a certain critical value of c standard deviations). The correlation exceeds the level is defined as the correlation between the two variables when they exceed standard deviations of their means, respectively:

$$\rho^{+}(c) = corr(r_{1t}, r_{2t} | r_{1t} > c, r_{2t} > c),$$
$$\rho^{-}(c) = corr(r_{1t}, r_{2t} | r_{1t} <- c, r_{2t} <- c),$$

where $c \geq 0$ is a given level, ρ^{+}(c) measures the correlation between two returns above a certain exceeding level c, ρ^{-}(c) measures the correlation between two returns below a certain exceeding level .

2.4. Distance correlation

Distance correlation is a statistic that measures the dependence between two random variables or data sets. It does not require the assumption of a linear relationship or distribution form between the variables. Unlike the traditional Pearson correlation coefficient, distance correlation can capture nonlinear relationships and is insensitive to changes in marginal distributions. Székely and Rizzo defined the distance-dependent statistic as follows [16].

Given an observed random sample $(\mathrm{U}, \mathrm{V}) = \{(\mathrm{U_i}, \mathrm{V_i}): \mathrm{i} = 1, \ldots, \mathrm{n}\}$ from the joint distribution of random vectors $\mathrm{U} \in \mathrm{R^p}$ and $\mathrm{V} \in \mathrm{R^q}$, the following is redefined:

$$c_{ij} = |U_i - U_j|_p, \quad \bar{c}_{i\cdot} = \frac{1}{n}\sum_{j=1}^{n} c_{ij}, \quad \bar{c}_{\cdot j} = \frac{1}{n}\sum_{i=1}^{n} c_{ij}, \quad \bar{c}_{\cdot\cdot} = \frac{1}{n^2}\sum_{i,j=1}^{n} c_{ij}$$
$$C_{ij} = c_{ij} - \bar{c}_{i\cdot} - \bar{c}_{\cdot j} + \bar{c}_{\cdot\cdot},$$

where i,j =1,...,n. Similarly, it is define:

$$d_{ij} = |V_i - V_j|_q, \text{ , then } D_{ij} = d_{ij} - \bar{d}_{i\cdot} - \bar{d}_{\cdot j} + \bar{d}_{\cdot\cdot},$$

for i,j =1,...,n. The empirical distance covariance $V_n(U,V)$ is the nonnegative number defined by:

$$\mathcal{V}_n^2(U,V) = \frac{1}{n^2}\sum_{i,j=1}^{n} C_{ij} D_{ij}.$$

Similarly, $V_n(U)$ is the nonnegative number defined by:

$$\mathcal{V}_n^2(U) = \mathcal{V}_n^2(U,U) = \frac{1}{n^2}\sum_{i,j=1}^{n} C_{ij}^2.$$

The empirical distance correlation $R_n(U,V)$ is the square root of

$$\mathcal{R}_n^2(U,V) = \begin{cases} \dfrac{\mathcal{V}_n^2(U,V)}{\sqrt{\mathcal{V}_n^2(U)\mathcal{V}_n^2(V)}}, & \mathcal{V}_n^2(U)\mathcal{V}_n^2(V) > 0; \\ 0, & \mathcal{V}_n^2(U)\mathcal{V}_n^2(V) = 0. \end{cases}$$

2.5. Chatterjee correlation

The Chatterjee correlation can better measure the individual asymmetry of correlation between financial risk assets. In recent years, Chatterjee introduced a new correlation coefficient calculation method, which mainly studies the correlation between individual assets [20]. Let (X,Y) be a pair of random variables where Y is not a constant. Let $(X_1,Y_2),\ldots,(X_n,Y_n)$ be iid pairs with the same law as (X,Y), where $n \geq 2$. Assume that X_i and Y_i have no relationship, and rearrange the data. Since X_i has no relationship, there is a unique way of doing this. Let τ_i be the rank of Y_i, that is, the number of j such that $Y_j \leq Y_i$. The new correlation coefficient is defined as

$$\xi_n(X, Y) := 1 - \frac{3\sum_{i=1}^{n-1} |Ti+1-Ti|}{n^2-1} \tag{3}$$

in the constrained case, ξn is defined as follows. If there are relationship among the Xi , then the incremental rearrangement as described above is chosen by breaking the relationship uniformly at random. Let τ_i be the same as before, and additionally define l_i to be the number of j such that $Y_j \geq Y_i$. Then define

$$\xi_n(X, Y) := 1 - \frac{n\sum_{i=1}^{n-1} |Ti+1-Ti|}{2\sum_{i=1}^{n-1} li(n-li)} \tag{4}$$

when there is no relationship among Y_i, $l_1, \ldots, l_n$, is just a permutation of 1,...,n, so the denominator in the above expression is just $n(n^2 - 1)/3$, which simplifies this definition to the Equation (3). Subsequently, several authors, through theoretical derivations and numerical simulations, demonstrated the advantages of Chatterjee's correlation coefficient over other commonly used measures of correlation, such as the Pearson correlation coefficient, under various statistical models [21]. This evidence shows that Chatterjee's correlation coefficient not only possesses theoretical optimality but also enhances performance in practical applications, particularly in high-dimensional data or settings involving complex dependencies.

3. New strategy for portfolio selection

Considering that the correlation between financial risk assets exhibits asym- metry in both time and individual dimensions, this section introduces new for- mulations to constructing the covariance matrix, which is then applied to derive more effective portfolio selection strategies. Three novel strategies is proposed using different combinations of correlation measurement tools:

(1) Shrinking the inverse covariance matrix to the product of inverse asymmetric correlation and inverse Chatterjee correlation (STICV-TVI).

(2) shrinking the inverse covariance matrix by combining the inverse asymmetric correlation with the inverse Chatterjee correlation (STICV(TV∪I)).

(3) Shrinking the inverse covariance matrix to a combination of the product of inverse asymmetric and Chatterjee correlations, plus the inverse covariance matrices of asymmetric and Chatterjee correlations (STICV(TV∪I∪TVI)).

3.1. Constructing the STICV-TVI

3.1.1. Step1: Construct covariance matrix

It is well established that the covariance matrix can be expressed in its variance correlation decomposition form, such that $\hat{\Sigma}_{\mathrm{d}} = \widehat{\Lambda}_{\mathrm{semi}}\widehat{\mathrm{R}}_{\mathrm{dis}}\widehat{\Lambda}_{\mathrm{semi}}$, where $\widehat{\Lambda}_{\mathrm{semi}}$ represents the semi-variance matrix, specifically $\mathrm{diag}(\widehat{\mathrm{SVAR}}_1, \ldots, \widehat{\mathrm{SVAR}}_{\mathrm{p}})$ and $\widehat{\mathrm{R}}_{\mathrm{dis}}$ signifies the distance correlation matrix, specifically $\mathrm{V}^2 = [\mathrm{V}_{\mathrm{ij}}^2]$, where $\mathrm{V}_{\mathrm{ij}}^2$ is the distance correlation between r_{it} and r_{jt}.

By leveraging the advantages of semi-variance and asymmetric correlation, replace the traditional variance and Pearson correlation matrix with the semi-variance matrix and the asymmetric correlation matrix, respectively. Consequently, the covariance matrix is expressed as $\hat{\Sigma}_{\mathrm{asi}} = \widehat{\Lambda}_{\mathrm{semi}}\widehat{\mathrm{R}}_{\mathrm{asi}}\widehat{\Lambda}_{\mathrm{semi}}$, where $\widehat{R}_{asy}$ represents the asymmetric correlation matrix, specifically $Z = [Z_{ij}]$, where $Z_{ij} = Z_{ji}$ and is computed based on this symmetric correlation. (iii) The individual correlation matrix is used as a replacement for the Pearson correlation matrix. Therefore, the covariance matrix can be defined as $\widehat{\Sigma}_{ast} = \widehat{\Lambda}_{semi}\widehat{R}_{asy}\widehat{\Lambda}_{semi}$, where $\widehat{R}_{asi}$ represents the individual correlation matrix, specifically $C = [c_{ij}]$, where $c_{ij} \neq c_{ji}$, each element C_{ij} is defined based on the dependency relationship between the corresponding variables according to Chatterjee, and the matrix is asymmetric.

For the convenience of parameter selection, an unknown parameter α needs to be introduced, so the following inverse sample covariance matrix can be built.

$$\widehat{S}_{\mathrm{TVI}}^{-1}1 = a\widehat{\Sigma}_{d}^{-1} + (1-a)\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1} \tag{5}$$

3.1.2. Step 2: Calculation of portfolio weights

Next, solve for the weights of the model using the equation below:

$$\widehat{\omega}(\widehat{S}_{\mathrm{TVI}}^{-1}) = \frac{\left(a\widehat{\Sigma}_{d}^{-1} + (1-a)\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)1}{1^{T}\widehat{S}_{\mathrm{TVI}}^{-1}1} \tag{6a}$$

Let us reintroduce an unknown coefficient β and let $\beta = \frac{a1^{T}\widehat{\Sigma}_{d}^{-1}1}{c}$. Through Equation (2) the equation below is obtained.

$$\widehat{\omega}(\widehat{S}_{\mathrm{TVI}}^{-1}) = \beta\widehat{\omega}\left(\widehat{\Sigma}_{d}^{-1}\right) + (1-\beta)\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1}\right) \tag{6}$$

From Equation (6), it is known that, to determine the optimal portfolio weights, calculate the coefficient.

3.1.3. Step 3: Selection of the shrinkage coefficients.

Using the variance minimization method to select the shrinkage coefficient β, the objective function can be derived as $\varphi_2\,\widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\right)^{\top}R_t + (1-\varphi_1-\varphi_2)\,\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\right)^{\top}R_t)$

$$var(\widehat{\omega}^{min}\left(\widehat{S}_{\mathrm{TVI}}^{-1}\right)^{T}\mathrm{R}_t) = \mathrm{var}(\beta\widehat{\omega}\left(\widehat{\Sigma}_{d}^{-1}\right)^{T}R_t + (1-\beta)\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{T}Rt \tag{7}$$

Simplify Equation (7), take its derivative with respect to and set it equal to 0, and then, find the solution as shown in Equation (8).

$$\beta = \frac{\mathrm{cov}(\widehat{\omega}\left(\widehat{\Sigma}_{d}^{-1}\right)^{T}R_t, \widehat{\omega}(\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1})^{T}R_t - var(\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{T}Rt)}{var(\widehat{\omega}\,(\widehat{\Sigma}_{d}^{-1})^{T}R_t - \widehat{\omega}(\widehat{\Sigma}_{asi}^{-1}\widehat{\Sigma}_{asi}^{-1})^{T}R_t} \tag{8}$$

3.2. Constructing the STICV(TV ∪ I)

The new model is constructed according to the following steps.

(1) Step 1: Construct the covariance matrix

Similar to step 1 in Section 3.1, for the convenience of parameter selection, introduce two unknown parameters , so the following model can be built

$$\widehat{S}_{\mathrm{TVUI}}^{-1} = \gamma_1\,\widehat{\Sigma}_{d}^{-1} + \gamma_2\,\widehat{\Sigma}_{ast}^{-1} + (1-\gamma_1-\gamma_2)\widehat{\Sigma}_{asi}^{-1} \tag{9}$$

(2) Step 2: Calculation of portfolio weights

Next, solve for the weights of the model, to get:

$$\widehat{\omega}(\widehat{S}_{\mathrm{TVUI}}^{-1}) = \frac{(\gamma_1\,\widehat{\Sigma}_{d}^{-1} + \gamma_2\,\widehat{\Sigma}_{ast}^{-1} + (1-\gamma_1-\gamma_2)\widehat{\Sigma}_{asi}^{-1})1}{1^{T}\widehat{S}_{\mathrm{TVUI}}^{-1}\,1} \tag{9a}$$

Let us reintroduce two unknown parameters φ 1, φ2 and let $\varphi_1 = \frac{\gamma_1 1\hat{\Sigma}_d^{-1}1}{c}$, $\varphi_2 = \frac{\gamma_2 1\hat{\Sigma}_{ast}^{-1}1}{c}$. So, $1-\varphi_1-\varphi_2 = \frac{(1-\gamma_1-\gamma_2)1^T\hat{\Sigma}_{asi}^{-1}1}{c}$. Through Equation (2), the below is obtained:

$$\hat{\omega}(\hat{S}_{\text{TV}\cup\text{I}}^{-1}) = \varphi_1\hat{\omega}(\hat{\Sigma}_d^{-1}) + \varphi_2\hat{\omega}(\hat{\Sigma}_{ast}^{-1}) + (1-\varphi_1-\varphi_2)\hat{\omega}(\hat{\Sigma}_{asi}^{-1}) \tag{10}$$

From Equation (10), it is know that it needs to find the generalized inverse weights of the three conditional covariances separately, namely $\hat{\omega}(\hat{\Sigma}_d^{-1})$, $\hat{\omega}(\hat{\Sigma}_{ast}^{-1})$ and $\hat{\omega}(\hat{\Sigma}_{asi}^{-1})$.

(3) Step 3: Selection of the shrinkage coefficients

Similarly to 3.1, use minimum variance method to select the shrinkage coefficients, and the objective function is as follows:

$$var\left(\hat{\omega}^{\min}\left(\hat{S}_{\text{TV}\cup\text{I}}^{-1}\right)^{\top}R_t\right) = var\Big(\varphi_1\hat{\omega}\left(\hat{\Sigma}_d^{-1}\right)^{\top}R_t + \varphi_2\hat{\omega}\left(\hat{\Sigma}_{ast}^{-1}\right)^{\top}R_t + (1-\varphi_1-\varphi_2)\hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\Big). \tag{11}$$

Simplify (11) first, then take the derivative of φ 1 and φ2 and set it equal to 0, and the result is as follow

$$\varphi = M^{-1}b, \tag{12}$$

Where φ = (φ1   φ2)T ,

φ = (φ1   φ2)T ,

$$M = \begin{bmatrix} A & B \\ B & C \end{bmatrix},$$

$$b = \begin{bmatrix} 2var\left(\hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) - 2cov\left(\hat{\omega}\left(\hat{\Sigma}_d^{-1}\right)^{\top}R_t, \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) \\ 2var\left(\hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) - 2cov\left(\hat{\omega}\left(\hat{\Sigma}_{ast}^{-1}\right)^{\top}R_t, \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) \end{bmatrix},$$

$R_t)$

where A,B,C are defined as follows

$$A = 2\text{var}\left(\hat{\omega}\left(\hat{\Sigma}_d^{-1}\right)^{\top}R_t - \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right), \quad C = 2\text{var}\left(\hat{\omega}\left(\hat{\Sigma}_{ast}^{-1}\right)^{\top}R_t - \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right),$$
$$B = 2\text{cov}\left(\hat{\omega}\left(\hat{\Sigma}_d^{-1}\right)^{\top}R_t, \hat{\omega}\left(\hat{\Sigma}_{ast}^{-1}\right)^{\top}R_t\right) - 2\text{cov}\left(\hat{\omega}\left(\hat{\Sigma}_d^{-1}\right)^{\top}R_t, \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) -$$
$$2\text{cov}\left(\hat{\omega}\left(\hat{\Sigma}_{ast}^{-1}\right)^{\top}R_t, \hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right) + 2\text{var}\left(\hat{\omega}\left(\hat{\Sigma}_{asi}^{-1}\right)^{\top}R_t\right).$$

3.3. Constructing the STICV(TV ∪ I ∪ TVI)

3.3.1. Step 1: Construct covariance matrix

Similar to step 1 in section 3.1., for the convenience of parameter selection, three unknown parameters α1, α2, α3 needs to be introduced, so the following model can be built:

$$\hat{S}_{\text{TV}\cup\text{I}\cup\text{TVI}}^{-1} = \alpha_1\hat{\Sigma}_d^{-1} + \alpha_2\hat{\Sigma}_{ast}^{-1} + \alpha_3\hat{\Sigma}_{asi}^{-1} + (1-\alpha_1-\alpha_2-\alpha_3)\hat{\Sigma}_{ast}^{-1}\hat{\Sigma}_{asi}^{-1} \tag{13}$$

3.3.2. Step 2: Calculation of portfolio weights

Next, solve for the weights of the model, to get:

$$\hat{\omega}(\hat{S}_{\text{TV}\cup\text{I}\cup\text{TVI}}^{-1}) = \frac{\left(\alpha_1\hat{\Sigma}_d^{-1} + \alpha_2\hat{\Sigma}_{ast}^{-1} + \alpha_3\hat{\Sigma}_{asi}^{-1} + (1-\alpha_1-\alpha_2-\alpha_3)\hat{\Sigma}_{ast}^{-1}\hat{\Sigma}_{asi}^{-1}\right)\mathbf{1}}{\mathbf{1}^{\top}\hat{S}_{\text{TV}\cup\text{I}\cup\text{TVI}}^{-1}\mathbf{1}}.$$

Reintroduce three unknown parameters β_1, β_2, β_3 and let $\beta_1 = \frac{\alpha_1 1^\top \hat{\Sigma}_d^{-1} 1}{c}$, $\beta_2 = \frac{\alpha_2 1^\top \hat{\Sigma}_{asi}^{-1} 1}{c}$, $\beta_3 = \frac{\alpha_3 1^\top \hat{\Sigma}_{ast}^{-1} 1}{c}$. So $1 - \beta_1 - \beta_2 - \beta_3 = \frac{(1-\alpha_1-\alpha_2-\alpha_3) 1^\top \hat{\Sigma}_{ast}^{-1} \hat{\Sigma}_{asi}^{-1} 1}{c}$. Next, simplify to get

$$\widehat{\omega}\left(\widehat{S}^{-1}_{\mathrm{TV}\cup\mathrm{I}\cup\mathrm{TVI}}\right) = \beta_1 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right) + \beta_2 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right) + \beta_3 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{asi}\right) + (1-\beta_1-\beta_2-\beta_3)\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right). \tag{14}$$

From Equation (14), it is known that it needs to find the generalized inverse weights of the three conditional covariances separately, namely $\widehat{\omega}(\widehat{\Sigma}_d^{-1})$, $\widehat{\omega}(\widehat{\Sigma}_{ast}^{-1})$ and $\widehat{\omega}(\widehat{\Sigma}_{asi}^{-1})$.

3.3.3. Step 3: Selection of the shrinkage coefficients

Similar to step 3 in section 3.1, the objective function is as follows:

$$var\left(\overset{\min}{\widehat{\omega}}\left(\widehat{S}^{-1}_{\mathrm{TV}\cup\mathrm{I}\cup\mathrm{TVI}}\right)^\top R_t\right) = var\Big(\beta_1 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t + (1-\beta_1-\beta_2-\beta_3)\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t + \beta_3 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t + \beta_2 \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right)^\top R_t\Big). \tag{15}$$

Simplify Equation 15, first, then take the derivative of β1, β2, and β3 and set it equal to 0, and the result is as

β = Q-1m,

where

β = (β1 , β2 , β3)T ,

$$Q = \begin{bmatrix} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \mathcal{B} & \mathcal{D} & \mathcal{E} \\ \mathcal{C} & \mathcal{E} & \mathcal{F} \end{bmatrix},$$

$$m = \begin{bmatrix} 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) \\ 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) \\ 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) \end{bmatrix},$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}$ are defined as follows

$$\mathcal{A} = 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t - \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right), \quad \mathcal{D} = 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right)^\top R_t - \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right),$$

$$\mathcal{B} = 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) + 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right),$$

$$\mathcal{C} = 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{d}\right)^\top R_t, \widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right) + 2var\left(\widehat{\omega}\left(\widehat{\Sigma}^{-1}_{ast}\widehat{\Sigma}^{-1}_{asi}\right)^\top R_t\right),$$

$$\mathcal{E} = 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\right)^{\top} R_t, \widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t\right) - 2cov\left(\widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\right)^{\top} R_t, \widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t\right) -$$

$$2cov\left(\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t, \widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t\right) + 2var\left(\widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t\right),$$

$$\mathcal{F} = 2var\left(\widehat{\omega}\left(\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t - \widehat{\omega}\left(\widehat{\Sigma}_{ast}^{-1}\widehat{\Sigma}_{asi}^{-1}\right)^{\top} R_t\right).$$

4. Data and methodology

4.1. Data

In the empirical analysis, the stock data employed comes from the Chinese stock market, to calculate its logarithmic return based on its closing price. For stock details, please see the **Table 1**.

Table 1. The data for 30 stocks are from the Chinese stock market from October 1, 2018 to March 18, 2020

Code	Code	Code	Code	Code
000568.SZ	002493.SZ	600276.SH	601012.SH	601933.SH
000651.SZ	300750.SZ	600346.SH	601288.SH	601939.SH
000708.SZ	300760.SZ	600438.SH	601318.SH	603259.SH
002024.SZ	600019.SH	600519.SH	601601.SH	603816.SH
000858.SZ	600028.SH	600690.SH	601628.SH	603833.SH
002304.SZ	600036.SH	600809.SH	601857.SH	603899.SH

4.1.1. Verify the asymmetry of correlation between individual stock returns

Section 2.3 mentioned the importance of asymmetric correlation in financial markets. To test the asymmetric correlation of the Chinese stock market dataset, the model-free test approach is used, formalizing the null hypothesis (H0: $\rho^+(c) = \rho^-(c)$ for all $c \geq 0$) and alternative hypothesis (HA: $\rho^+(c) \neq \rho^-(c)$ for some $c \geq 0$) of Hong *et al.* [12]. The significance level of the hypothesis test is 0.05, and c is a given level (c = 0 in this paper). So the statistic for testing the null hypothesis is defined as follows:

$$J_p = T(\hat{\rho}^+ - \hat{\rho}^-)^{\top}\hat{O}^{-1}(\hat{\rho}^+ - \hat{\rho}^-),$$

Where

$$\hat{O} = \sum_{l=1-T}^{T-1} k(l/p)\hat{\Upsilon}_l, \quad k(\cdot) = (1-|z|)I(|z|<1), \quad \hat{\Upsilon}_l(c_i, c_j) = \frac{1}{T}\sum_{t=|l+1|}^{T} \hat{\chi}_t(c_i)\hat{\chi}_{t-|l|}(c_j),$$

$$\hat{\chi}_t(c) = \frac{T}{T_c^+}\left[\hat{X}_{1t}^+(c)\hat{X}_{2t}^+(c) - \hat{\rho}^+(c)\right] I(r_{1t} > c, r_{2t} > c) -$$

$$\frac{T}{T_c^-}\left[\hat{X}_{1t}^-(c)\hat{X}_{2t}^-(c) - \hat{\rho}^-(c)\right] I(r_{1t} < -c, r_{2t} < -c),$$

where T is the size of sample, and I is the indicator function, Υ_l is a matrix with (i, j)-th element, and $k(\cdot)$ is a kernel function that assigns a suitable weight to each lag of order *l*, *p* is the smoothing parameter or lag truncation order. In this paper, the Bartlett kernel is used.

Table 2. The partial *P*-value of stock pair

Stock pair	*P*-value	Stock pair	*P*-value
(000858.SZ, 002493.SZ)	5.20E-03	(002304.SZ, 002493.SZ)	1.34E-04
(600438.SH, 600519.SH)	4.15E-02	(601939.SH, 603259.SH)	5.06E-04
(603259.SH, 000568.SZ)	9.40E-03	(601857.SH, 603259.SH)	6.20E-04
(002493.SZ, 601012.SH)	3.21E-02	(002493.SZ, 002024.SZ)	7.66E-05
(601012.SH, 603259.SH)	1.40E-03	(603259.SH, 601318.SH)	3.86E-04

Table 2 presents the partial *P*-values of the stock pairs under analysis. Setting the significance level for the hypothesis test at 0.05, the analysis of the data in **Table 2** reveals that the calculated *P*-values are significance lower than the set threshold. This finding provides robust evidence to support the rejection of the null hypothesis, thus favoring the acceptance of the alternative hypothesis. Consequently, it can be inferred that there is sufficient evidence to demonstrate the existence of asymmetric correlation between pairs of stocks.

4.1.2. Verify the Chatterjee correlation between individual stock returns

The returns of eight stocks (**Table 3**) is used to verify the Chatterjee correlation. **Table 4** presents the partial of the stock pairs under analysis. Due to Moutai's market leadership and brand in influence in the alcohol industry, take Kweichow Moutai's stock code (600519.SH) as an example of the leading stocks compared to Shanxi Fenjiu (600809.SH). It is found that within the same industry, 600519.SH's in influence on 600809.SH is greater than 600809. SH's in influence on 600519.SH, and the situation is similar in other industries, such as Sinopec Corp. (600028.SH) and Hengli Petrochemical (600346.SH). There exists asymmetry of correlation between stocks, and that Chatterjee correlation can effectively measure the asymmetry of correlation between stocks individuals.

Table 3. Select eight of these stocks to verify Chatterjee Correlation

Code	English abbreviation
600809.SZ 600519.SH 600346.SH 600028.SH	Shanxi Xinghuacun Fen Wine Factry Co Ltd Kweichow Moutai Co. Ltd Hengli Petrochemical Co Ltd China Petroleum Chemical Corp (Sinopec Corp.)

Table 4. The partial ξn (X, Y) of stock pair

Code	600809.SH	600519.SH	Code	600028.SH	600346.SH
600809.SH	0.9925	0.0482	600028.SH	0.9925	0.0437
600519.SH	0.1124	0.9925	600346.SH	-0.0214	0.9925

4.2. Portfolio selection strategies

Analyze the proposed portfolio strategy by comparing it with several established strategies documented in the literature (**Table 5**). The strategies LW(id) and LW(lf) are derived from the research of Ledoit, Wolf [22, 23]. The ICVARI, ICVARF, and ICVARIF strategies are based on the work of Kourtis *et al*. [24].

Table 5. List of portfolio selection strategies

Abbreviation	Expression
Panel A: The benchmark optimization model for this portfolio selection	
ICVARF ICVARIF ICVARI LW(lf) LW(id) AST·ASI ASI NAIVE GMV	$\hat{S}^{-1}_{ARF} = \alpha \hat{\Sigma}^{-1}_{ast} + (1-\alpha) \hat{\Sigma}_F$ $\hat{S}^{-1}_{ARIF} = \alpha_1 \hat{\Sigma}^{-1}_{ast} + \alpha_2 \hat{\Sigma}_I + (1-\alpha_1-\alpha_2) \hat{\Sigma}_F$ $\hat{S}^{-1}_{ARI} = \alpha \hat{\Sigma}^{-1}_{ast} + (1-\alpha) \hat{\Sigma}_I$ $\hat{S}_{lf} = \alpha \hat{\Sigma}_{ast} + (1-\alpha) \hat{\Sigma}_F$ $\hat{S}_{id} = \alpha \hat{\Sigma}_{ast} + (1-\alpha) \hat{\Sigma}_I$ $\hat{S}^{-1}_{TI} = \hat{\Sigma}^{-1}_{ast} \hat{\Sigma}^{-1}_{asi}$ $\hat{S}^{-1}_{ASI} = \hat{\Sigma}^{-1}_{asi}$ $\hat{S}^{-1}_{NA} = (1/p) I$ $\hat{S}^{-1}_{GMV} = \hat{\Sigma}^{-1}$-1
Panel B: The new optimization model	
STICV-TVI STICV(TV ∪ I) STICV(TV ∪ I ∪ TVI)	$\hat{S}^{-1}_{TVI} = \alpha \hat{\Sigma}^{-1}_{d} + (1-\alpha) \hat{\Sigma}^{-1}_{ast} \hat{\Sigma}^{-1}_{asi}$ $\hat{S}^{-1}_{TV \cup I} = \gamma_1 \hat{\Sigma}^{-1}_{d} + \gamma_2 \hat{\Sigma}^{-1}_{ast} + (1-\gamma_1-\gamma_2) \hat{\Sigma}^{-1}_{asi}$ $\hat{S}^{-1}_{TV \cup I \cup TVI} = \alpha_1 \hat{\Sigma}^{-1}_{d} + \alpha_2 \hat{\Sigma}^{-1}_{ast} + \alpha_3 \hat{\Sigma}^{-1}_{asi}$ $+ (1-\alpha_1-\alpha_2-\alpha_3) \hat{\Sigma}^{-1}_{ast} \hat{\Sigma}^{-1}_{asi}$

4.3. Performance evaluation metric

To evaluate out-of-sample portfolio performance, the rolling window technique is employed. Specifically, the portfolio weight ω_t^k for each strategy *k* was estimated using the daily returns from *t-h* to *t*-1, where *h* represents the window length. Subsequently, the corresponding out-of-sample portfolio returns at time *t*+1 were calculated as $R_{t+1}^k = (\omega_t^k)^\top \cdot R_{t+1}$, producing a time series of excess returns for each portfolio strategy. Here, R_{t+1} denotes the asset returns at time *t*+1. The sample mean $\hat{\mu}^k$ and standard deviation $\hat{\sigma}^k$ of the excess returns were then computed, followed by the calculation of the out-of-sample performance metrics as outlined in **Table 6**.

Table 6. Partial results for out-of-sample Mean

Name	Expression
Mean	$\hat{\mu}^k = \frac{1}{n-h} \sum_{t=h}^{n-1} (\omega_t^k)^\top R_{t+1}$
Sharpe ratio	Sharpe ratio $\widehat{SR}^k = \frac{\hat{\mu}^k}{\hat{\sigma}^k}$, with $\hat{\sigma}^k = \sqrt{\frac{1}{n-h-1} \sum_{t=h}^{n-1} \left((\omega_t^k)^\top R_{t+1} - \hat{\mu}^k\right)^2}$

5. Empirical results

In this study, 30 stocks from the Chinese stock market is examined. To create datasets of varying dimensions, 5, 10, 15, and 20 stocks is randomly selected from the original 30. For each subset, calculate the returns. The subsequent analysis focuses on two key aspects: Average Return and Sharpe Ratio (SR).

5.1. Results for out-of-sample mean

Table 7 presents the results for the average excess partial returns for each portfolio strategy examined (The data results in the table are also enlarged 1000 times). From the experimental data, the following conclusions can be drawn. Comparing with other indicators, it is found that STICV-TVI, STICV(TVUI), and STICV(TVUIUTVI) out-of-sample mean is generally better than the other indicators; there is a higher out-of-sample mean, which means that asymmetric correlation, individual structure, and time-varying structure are useful, especially when combined with the distance correlation matrix. The results indicate that the combination of multiple correlation coefficients has a positive effect on increasing the out-of-sample mean of the portfolio.

Table 7. Partial results for out-of-sample Mean

h	Method	5IP	10IP	15IP	20IP
	ICVARF	0.5525	1.0941	0.2929	1.0084
	ICVARIF	0.2329	0.5486	0.8669	0.8371
	ICVARI	1.1062	1.4118	1.6999	1.1942
	LW(lf)	0.3055	0.3999	0.8072	0.2750
	LW(id)	0.6875	1.2797	1.0306	1.0051
	AST·ASI	1.2186	4.8612	-2.9773	1.9278
	ASI	0.4567	0.7216	0.7562	0.5812
20	NAIVE	0.5990	1.0360	1.0835	0.7189
	GMV	0.2256	0.5149	0.9616	-0.0058
	STICV(TVUI)	-7.9213	1.6049	-3.5750	8.8302
	STICV(TVUIUTVI)	2.5275	0.9527	2.2463	0.6912
	STICV-TVI	1.6241	-0.3569	8.2101	0.5884

5.2. Results for out-of-sample Sharpe ratio

Table 8 shows the partial results for reports. The average SR of the excess returns for each portfolio strategy considered and the data results in the table are also enlarged 1000 times. From the experimental data, the following conclusions can be drawn.

Regardless of the length of the window width and the dimension of the portfolio, STICV (TVUIUTVI) outperforms other indicators in terms of Sharpe ratio performance. This shows that the shrinkage method and asymmetry used in the model effectively reduce the estimation error and improve the effectiveness of risk management. At the same time, adding individual structure and time-varying structure factors to the model more accurately captures the correlation and volatility of assets. The combination of these methods also enables the portfolio to better balance risk and return. If the results of STICV-TVI, STICV(TVUI), and STICV (TVUIUTVI) is compared, it is found that STICV (TVUIUTVI) is better than STICV-TVI and STICV(TVUI).

Table 8. Partial results for out-of-sample SR

h	Method	5IP	10IP	15IP	20IP
	ICVARF	14.7382	34.3888	0.9208	3.5457
	ICVARIF	6.1908	16.5571	2.7435	2.9913
	ICVARI	60.3090	97.7646	35.5108	26.4221
	LW(lf)	18.0527	17.4015	37.6193	9.9798
	LW(id)	34.3661	59.3272	54.3114	41.7454
	AST·ASI	38.3274	45.9780	-6.6462	40.3556
	ASI	25.7129	46.7267	43.8700	36.2112
20	NAIVE	29.4312	66.6518	22.2036	15.5550
	GMV	12.3570	25.4235	46.9831	-0.2464
	STICV(TVUI)	-42.1990	26.0394	-39.8212	41.8262
	STICV(TVUIUTVI)	80.2958	50.8548	71.4911	29.6349
	STICV-TVI	77.6193	-17.6901	54.2835	19.6410

6. Conclusion

This paper investigates the application of asymmetric correlation and its extensions to portfolio selection problems. By incorporating both time and individual dimensions of asymmetric correlation, a robust and effective asymmetric influence matrix is developed. This matrix is applied within the compressed inverse covariance matrix portfolio selection framework to optimize corresponding parameters. The results demonstrate that these innovations enhance portfolio performance under the GMV model, offering practical insights for investors.

The proposed portfolio selection strategy is applied to empirical data. Empirical analysis reveals that: (1) applying only the time or individual dimension with the standard correlation coefficient matrix is insufficient for optimal performance; (2) after adopting the new strategy STICV (TVUIUTVI), achieves a higher out-of-sample mean and outperforms traditional approaches in terms of the Sharpe ratio. However, this study also highlights challenges. While shrinking the inverse covariance matrix significantly improves portfolio performance, limited research exists on shrinking the inverse of asymmetric correlation matrices. This paper uses the generalized inverse to address this, but recognizes the need for further exploration into simplifying parameter calculations within this framework. Future research should aim to refine covariance matrix shrinking techniques under asymmetric influence conditions.

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