

Theoretical Study and Slip Effect Analysis of Elastic Calculation Methods for Steel-Concrete Composite Beams

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Abstract: Steel-concrete composite beams, due to their superior mechanical properties, are widely utilized in engineering structures. This study systematically investigates the calculation methods for internal forces and load-bearing capacity of composite beams based on elastic theory, with a focus on the transformed section method and its application under varying neutral axis positions. By deriving the geometric characteristics of the transformed section and incorporating a reduction factor accounting for slip effects, a computational model for sectional stress and ultimate load-bearing capacity is established. The results demonstrate that the slip effect significantly influences the flexural load-bearing capacity of composite beams. The proposed reduction factor, which considers the influence of the steel beam's top flange thickness, offers higher accuracy compared to traditional methods. These findings provide a theoretical foundation for the design and analysis of composite beams, with significant practical engineering value.

Keywords: Composite beam; Elastic calculation; Slip effect; Theoretical study

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1. Introduction

Steel-concrete composite beams, leveraging the high strength of steel and the excellent compressive properties of concrete, are extensively applied in bridges, buildings, and other structural systems. In recent years, extensive research has been conducted on calculation methods for composite beams, leading to the development of theoretical models such as the transformed section method, interpolation method, analytical method, and reduced stiffness method. Among these, the transformed section method is widely adopted due to its simplicity and engineering applicability. However, traditional transformed section methods often neglect the slip effect at the

steel-concrete interface, resulting in overestimated load-bearing capacity calculations that fail to accurately reflect actual stress conditions.

This study systematically analyzes the application of the transformed section method for calculating internal forces and load-bearing capacity of composite beams based on elastic theory. The analysis addresses two cases: the neutral axis located within the steel beam and within the concrete flange. Corresponding formulas for sectional geometric characteristics and stress distribution are derived. Additionally, a reduction factor accounting for the slip effect, incorporating the influence of the steel beam's top flange thickness, is proposed to enhance the accuracy of load-bearing capacity calculations. This research aims to provide a more reliable theoretical basis for composite beam design, addressing the limitations of traditional methods in handling slip effects.

2. Basic theory

According to the calculation principles of elastic theory, the stress and stiffness of composite beams are calculated using the methods of material mechanics. Therefore, the conversion section is adopted to convert the concrete section into the steel section ^[1-3].

Suppose there is a steel bar plate at a certain height of the concrete slab. From basic assumption (2), it can be known that $\varepsilon_c = \varepsilon_s$. Then, from basic assumption (1), the stress at a certain height of the concrete slab can be obtained as:

$$\sigma_c = \varepsilon_c E_c \quad (1)$$

Since $\varepsilon_s = \frac{\sigma_s}{E_s}$, where σ_s is the stress of the steel strip, then

$$\sigma_c = \varepsilon_c E_c = \frac{\sigma_s}{E_s} E_c = \frac{1}{\alpha_{ES}} \sigma_s \quad (2)$$

In the formula, σ_c , σ_s represent the compressive stresses of concrete and steel plate, respectively.

ε_c , ε_s respectively represent the strains of concrete and steel strip at the same height on the cross-section;

E_c , E_s respectively represent the elastic modulus of concrete and that of steel;

α_{ES} represents the ratio of the elastic modulus of steel to that of concrete, $\alpha_{ES} = E_s/E_c$.

Based on the fact that the position and magnitude of the resultant force application point remain unchanged, convert the area A_c of the concrete wing plate to the equivalent converted area A_{cs} of the steel.

$$A_{cs} = \frac{1}{\alpha_{ES}} A_c \quad (3)$$

Its physical meaning is: under the conditions of equal strain and constant total internal force, by dividing the area A_c of the concrete by α_{ES} , the area A_c of the concrete slab can be converted into the equivalent cross-sectional area A_{cs} of the steel.

In order to keep the centenary height of the concrete section unchanged before and after conversion, the converted section thickness of the concrete slab remains the same as the original section thickness; that is, only the width of the concrete slab is converted. If the calculated width b_e of the concrete bridge deck of the composite beam.

$$b_e = b_0 + b_{c1} + b_{c2}$$

In the formula, b_0 represents the width of the upper flange of the steel beam;

b_{c1} , b_{c2} are the calculated widths of the concrete wing plates measured on the outside and inside of the beam, taken according to the following minimum values: 1/6 of the beam span L , 1/2 of the distance s between the beams, and the actual overextension length of the concrete plate.

Then the converted width of the board is

$$b_{eq} = \frac{1}{\alpha_{ES}} b_e \quad (4)$$

Based on the above principle of conversion section, the section of the steel-concrete composite beam can be converted into an equivalent section of the steel beam, and then the geometric characteristics of the conversion section can be calculated according to the converted section of the steel beam, as shown in **Figure 1**.

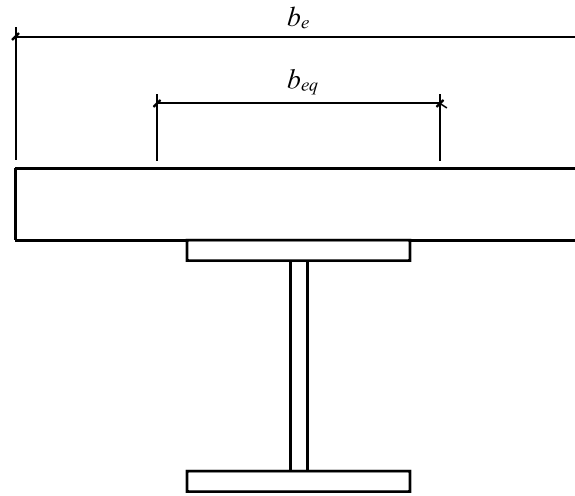


Figure 1. Conversion section of composite beams

3. Calculation method for section resistance distance

After obtaining the geometric characteristics of the converted section, the section stress and ultimate bearing capacity of the composite beam can be directly calculated by using the relevant formulas of material mechanics. Calculate the geometric characteristics of the conversion section respectively for the two cases where the neutralization axis is within the concrete wing plate and outside the concrete plate ^[4,5].

3.1. The neutralizing shaft is inside the steel beam

The situation where the neutral axis of the composite beam is located within the web of the steel beam is shown in **Figure 2**.

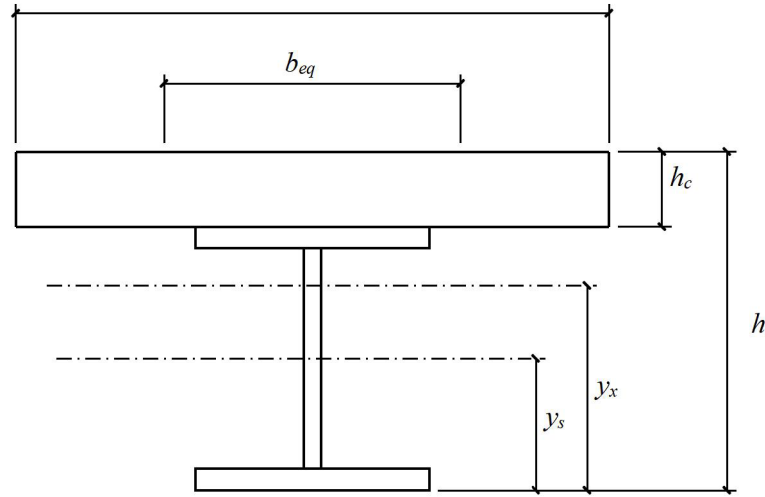


Figure 2. The composite beam and shaft are inside the steel beam

Convert the area of the cross-section A_0

$$A_0 = A_s + A_{cs} \quad (5)$$

Convert the cross-section and the wheelbase distance (with the lower flange edge of the steel beam as the baseline) y_x

$$y_x = \left[A_s y_s + A_{cs} \left(h - \frac{h_c}{2} \right) \right] / A_0 \quad (6)$$

Convert the moment of inertia I_0 of the cross-section

$$I_0 = \frac{1}{12} b_{eq} h_c^3 + A_{cs} \left(h - y_x - \frac{h_c}{2} \right)^2 + I_s + A_s (y_x - y_s)^2 \quad (7)$$

The cross-sectional resistance moment at the edge of the lower flange of the steel beam

$$W_{0s}^b = \frac{I_0}{y_x} \quad (8)$$

The cross-sectional resistance distance of the upper edge of the concrete

$$W_{0c}^t = \frac{I_0}{h - y_x} \quad (9)$$

3.2. The neutralizing axis is located within the concrete wing plate

The situation where the central axis of the composite beam is located within the concrete wing plate is shown in **Figure 3**.

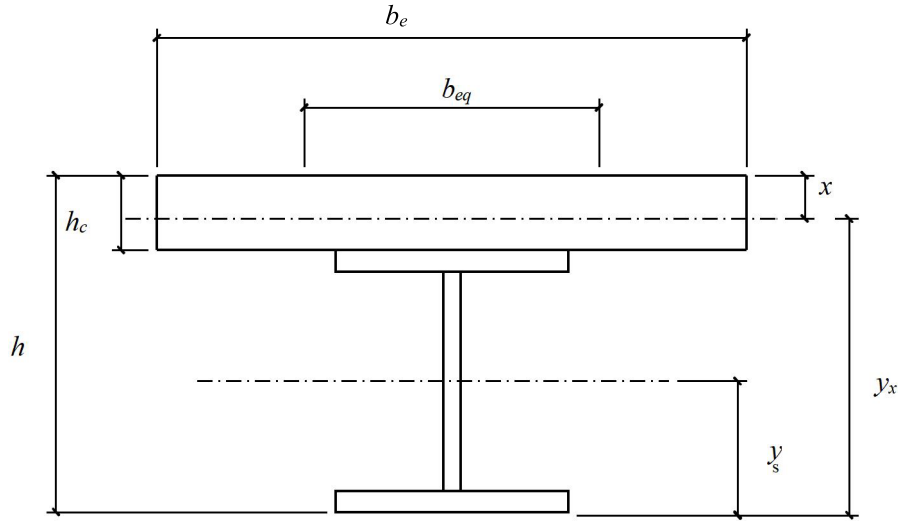


Figure 3. Composite beam and shaft inside the concrete wing

Firstly, based on the area-moment equilibrium equation around the neutral axis, the distance x between the neutral axis and the upper edge of the composite beam can be solved

$$\frac{1}{2}b_{eq}x^2 - A_s(h - y_s - x) = 0 \quad (10)$$

If the concrete in the tensile zone is ignored, then convert the area of the cross-section

$$A_0 = b_{eq}x + A_{cs} \quad (11)$$

Convert the cross-section and the wheelbase distance (taking the lower flange edge of the steel beam as the baseline)

$$y_x = h - x \quad (12)$$

Convert the moment of inertia I_0 of the cross-section

$$I_0 = \frac{1}{3}b_{eq}x^3 + I_s + A_s(h - y_s - x)^2 \quad (13)$$

The cross-sectional resistance moment at the edge of the lower flange of the steel beam

$$W_{0s}^b = \frac{I_0}{y_x} \quad (14)$$

The cross-sectional resistance distance of the upper edge of the concrete

$$W_{0c}^t = \frac{I_0}{x} \quad (15)$$

4. Analysis of the impact of the slip effect

4.1. The ultimate bearing capacity of the composite beam and the normal internal force of the section during sliding are not considered

The section resistance moment of the converted section was obtained earlier. Then, the elastic ultimate bearing

capacity is calculated as

$$M_y = W_{0s}^b f_y \quad (16)$$

The normal stress of the cross-section at the edge of the lower flange of the steel beam

$$\sigma_{0s}^b = \frac{M}{W_{0s}^b} \quad (17)$$

The normal stress of the cross-section at the upper edge of the concrete wing slab

$$\sigma_{0c}^t = \frac{M}{\alpha_{ES} W_{0c}^t} \quad (18)$$

4.2. Consider the ultimate bearing capacity of the composite beam and the normal internal force of the section during sliding

In the elastic calculation, the slip between the steel beam and the concrete flange was ignored. However, in actual engineering, due to the existence of slip, the actual elastic flexural bearing capacity of the section is less than that obtained by the converted section method. Under the same bending moment, the normal stress of the section considering slip will also be greater than the calculation result obtained by the converted section method^[6,7].

If other assumptions remain unchanged, considering slip, Hypothesis (2) is changed to indicate that there is relative slip between the steel beam and the concrete wing plate. However, it is still assumed that their curvatures are the same, and the additional stress caused by slip strain is linearly distributed, as shown in **Figure 4**.

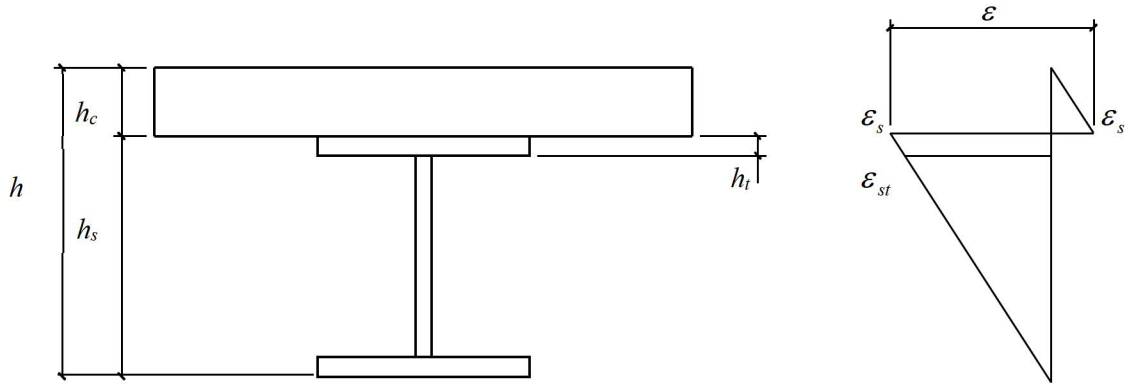


Figure 4. Additional strain caused by slip strain

The dimensions and strains of each part of the composite beam are shown in **Figures 2–4**. From the assumption, the relationship between the additional curvature and strain is as follows

$$\Delta\varphi = \frac{\varepsilon}{h} = \frac{\varepsilon_s}{h_s} = \frac{\varepsilon_c}{h_c} = \frac{M\zeta}{EI} \quad (19)$$

In the formula, ε is the relative slip strain; ε_s and ε_c are the slip strains at the junction of the steel beam and the concrete flange, respectively. ζ is the stiffness reduction coefficient; EI is the stiffness of the converted section.

The strain at the junction of the web and the upper flange of the steel beam is

$$\varepsilon_{st} = \frac{h_s - h_t}{h_s} \varepsilon_s \quad (20)$$

The additional resultant force of the web of the steel beam is

$$\Delta N_1 = \frac{1}{2} E_s \varepsilon_{st} A_w \quad (21)$$

In the formula, A_w represents the web area of the steel beam

The additional resultant force of the flange plate on the steel beam is

$$\Delta N_2 = \frac{1}{2} E_s (\varepsilon_{st} + \varepsilon_s) A_t \quad (22)$$

In the formula, A_t represents the area of the upper flange of the steel beam

Then the additional bending moment of the cross-section is

$$\Delta M = \Delta N_1 d_1 + \Delta N_2 d_2 \quad (23)$$

In the formula, d_1 and d_2 are respectively the distances from the points of application of the additional resultant force of the web and upper flange of the steel beam to the points of application of the additional resultant force of the concrete.

$$d_1 = \frac{1}{3} (h_s - h_t) + h_t + \frac{1}{3} h_c = \frac{1}{3} (h + 2h_t) \quad (24)$$

$$d_2 = \frac{1}{2} h_t + \frac{1}{3} h_c \quad (25)$$

Substitute to obtain the additional bending moment

$$\Delta M = \frac{E_s M_s \xi}{6EI} \left[(h_s - h_t) (h + 2h_t) A_w + \left(h_s - \frac{h_t}{2} \right) (3h_t + 2h_c) A_t \right] \quad (26)$$

Then the actual bending moment of the composite beam is

$$M_p = M - \Delta M \quad (27)$$

Let $M_p = \beta M$, then the slip reduction coefficient β has the following expression

$$\beta = 1 - \frac{h_s E_s \xi}{6EI} \left[\left(1 - \frac{h_t}{h_s} \right) (h + 2h_t) A_w + \left(h_s - \frac{h_t}{2} \right) (3h_t + 2h_c) A_t \right] \quad (28)$$

Calculation of normal stress in cross-section

$$\sigma = \frac{M - \Delta M}{W} = \frac{\beta M}{W} \quad (29)$$

In the formula, W represents the corresponding cross-sectional resistance distance obtained by the conversion section method.

The flexural bearing capacity in the elastic limit state is

$$M_{py} = \beta M_y \quad (30)$$

When conducting elastic theory analysis, the influence of slip was taken into account. Through the calculation of the additional bending moment, the reduction coefficient of the elastic ultimate flexural bearing capacity considering slip was obtained. The reduction coefficient derived in this paper takes into account the influence of the thickness of the upper flange of the steel beam, and the result is more accurate.

5. Conclusion

This paper provides an in-depth study of the elastic calculation method for steel-concrete composite beams, systematically explaining the theoretical foundation of the equivalent section method and its application at different neutral axis positions. In addressing the impact of interface slip on bearing capacity, a reduction factor

considering the thickness of the upper flange of the steel beam is derived, significantly improving the calculation accuracy. The research results indicate that the slip effect cannot be overlooked, and its influence on the bending capacity and section stresses of the composite beam must be fully considered in the design. The theoretical derivations and calculation models presented in this paper provide an important reference for the engineering design and analysis of composite beams. Future work could involve further validation of the model's applicability through experimental data and the exploration of computational methods for the nonlinear stage, expanding its application under complex conditions.

Disclosure statement

The authors declare no conflict of interest.

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