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Global Terminal Switching Sliding Mode Control of Robot Manipulators Based on Entire Fixed-Time Disturbance Observer

Fengning Zhang*

College of Physics and Electronic Information Engineering, Jining Normal University, Ulanqab 012000, Mongolia, China

**Author to whom correspondence should be addressed.*

Abstract: This paper presents an entire fixed-time disturbance observer-based global terminal switching sliding mode control of robot manipulators, which has inner and external uncertainties. The entire fixed-time disturbance observer-based global terminal switching sliding mode control has the global finite-time reaching characteristic, the property that system convergence time can be prescribed, and the global robustness to uncertainties, with the entire fixed-time disturbance observer that accurately estimates uncertainties after a fixed time, despite the initial state. The joints of the control system can arrive at the prescribed joint angular position at the predefined joint angular speed at the prescribed time.

Keywords: Prescribed-time stability; Sliding mode; Switched control

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1. Introduction

Switched control (SC) and sliding mode control (SMC) are adopted in the control of robot manipulators [1]. High-speed non-singular terminal (HNT) switched SMC (SSMC) method for robotic arms is proposed in the literature [1]. This method switches sliding mode controllers, according to the requirements, to improve the performance of the control system. HNT switched sliding mode (HNT-SSM), providing global non-singularity, which HNT-SSMC executes, represents different control requirements. And HNT-SSM exhibits global high-speed convergence. Simulation studies were conducted on application examples. SC has attracted attention owing to its effectiveness in enhancing performance [2]. This paper studies the design of state feedback SC in switched discrete-time systems [2]. The objective of the paper design the function and parameters, which ensure H_2 and H_∞ performance. The inequalities are the basis of this prerequisite, which permits the derivation under the fixed scalar variable, which is expressed as Linear Matrix Inequalities. The theoretical results are well-suited for addressing the control problem, where the switching rule is designed to improve performance in the channel. This approach is compared with methods in other papers. Examples in switched and network control systems validate the proposed technology.

SMC is well known for its merits, such as simplicity, good transient process, and robustness to uncertainties [3]. In the literature, in traditional SMC systems, sliding movement is reduced-order [3]. Double issues are hindering the practical application of SMC, which are the singularity in terminal SMC systems and the chattering of SMC systems, including traditional linear SMC and terminal SMC systems. The full-order terminal SMC method without chattering is presented in this paper. Because the control law does not contain derivatives concerning fractional powers, the singularity is averted. The continuous SMC law is proposed to achieve chatter-free SMC. The system shows a full-order dynamic, instead of an ideal reduced-order dynamic, in a sliding motion process. The design technique of the proposed full-order SMC for nonlinear systems, which can solve both problems of chatter and singularity, is presented. The presented chatter-free SMC is verified by the simulation. SMC could be found in many practical applications. SMC drives and maintains the state of the sliding manifolds, which are designed according to control demands. The control system has the invariance for any uncertainty in the sliding mode. For conventional SMC, the convergence time is infinite owing to the asymptotic stability of the linear sliding manifold.

Non-singular terminal sliding-mode (NTSM) control offers finite-time convergence [4]. This paper proposes an NTSM controller for systems which has inner and external uncertainties [4]. The total time, concerning the time that is needed to reach the sliding manifold and the time that is needed to reach the equilibrium in the sliding manifold, is proved to be finite. The presented novel sliding manifold eliminates the singularity issues in traditional terminal SMC. The control approach is applied to control rigid robotic arms. Simulation verifies the correctness of the analysis. Because of the advances in microprocessor technology, the presented SMC algorithm can control the actual robots, since variables with fractional exponents are embedded into the proposed law in microprocessors without difficulty. However, NTSM control cannot provide global robustness due to its reaching phase.

Time-varying SMC presents invariance to disturbance, i.e., global robustness [5]. A SMC technique for the second-order nonlinear systems with bounded inputs is proposed in this paper [5]. There are three sliding mode (SM) presented: The first two use moving SM, and the last one utilizes a terminal SM. All three SM in the beginning pass via representative points of the system, then move to the equilibrium within a finite time. This method does not have the reaching phase of traditional SMC, and ensures the invariance of the system concerning inner and external uncertainties from the beginning of the motion. The integral of the absolute counts of the errors is minimized by the design method for the time-varying sliding lines. Theoretical analysis and simulation results show that the designed law presented in the literature has a better transient response concerning the traditional SM. The time-varying SM shows particularly good properties and ensures that the state of the error system converges to the equilibrium. Nonetheless, the convergence time of terminal SM control is infinite. A paper presents an approach to combine the desired time convergence and the global robustness [6]. This paper provides a new design method for nonlinear systems with inner and external uncertainties [6]. By introducing a new function concerning time into the existing sliding manifold, a finite-time sliding manifold is presented, and then the paper presents a new time-varying terminal SMC. The control approach completely does not have the reaching phase, and guarantees that the system state is in the sliding manifold throughout the process. Besides these, the whole movement process and the accurate convergence time can be predicted, a method of determining parameters is provided to ensure that the convergence time can be precisely decided. The simulation results of a dynamic system are presented to validate the proposed approach. The paper does not cover the reaching condition, which is important to SMC and its application [6,7]. This paper introduces a tutorial on sliding mode variable structure control [7]. The aim is to briefly introduce the theoretics, main findings, and applications of this potent controller design strategy. This method is especially suitable for the controller design of uncertain systems. The outstanding features of this control system design approach, such as invariance and chattering, are focused. Methods to address chattering are provided. The linear systems and the nonlinear systems are included simultaneously. Finally, the paper gives future research directions, and a large number of papers are listed.

Finite-time disturbance observers (FDO) improve the performance of controllers [8,9]. In the literature, the preset finite-time stability issue for the uncertain SISO systems that is with inner and external uncertainties and asymmetric input constraints [8]. The immeasurable external disturbance is approximated by the designed, prescribed FDO. At the same time, a nonsingular preset finite-time SM controller design method is presented to control the asymmetric input-constrained system. The control system's state and unknown parameters are accurately estimated by the Kalman approach in this paper. Furthermore, this paper uses particle swarm optimization to get the parameters of the FDO and the controller. The scheme is used to ensure preset finite-time stabilization of nonlinear vibrations in non-local strain gradient nanobeams. Finally, through numerical simulation, the developed adaptive control scheme is compared with traditional sliding mode control to demonstrate its effectiveness and performance in nanobeam vibration control. This literature presents an integral finite-time SM controller based on FDO to achieve a good control characteristic of the rectifier [9]. The mathematical model is founded on time-varying nonlinear state equations. The fluctuation of the DC load is considered uncertain. The association generates the reference for the current loop. The effectiveness of the control system is validated by numerical simulations, and it can be found that the system state converges to the preset values under the existence of inner and external uncertainties. The integral finite-time SM controller maintains the robustness to uncertainties and fast transient response of the device. AC power with the harmonics is assumed to verify the good characteristics of the control approach presented. The real-time experiment is completed, and the good characteristics of the presented control approach have been validated. Nevertheless, the FDO does not have the characteristic of entire fixed-time convergence—the bigger the absolute value of the difference between the initial position and the equilibrium, the more time the system state takes to converge to the equilibrium, till infinity.

The objective of this paper is to propose an entire fixed-time disturbance observer-based global terminal switching sliding mode control (EFDO-GTSSMC) of robot manipulators. The EFDO that precisely approximates uncertainties after a fixed time, regardless of the initial state, is first proposed. Based on EFDO, the EFDO-GTSSMC is presented, which has the global finite-time reaching characteristic, the property that system convergence time can be prescribed, and the global robustness to uncertainties. By using the EFDO-GTSSMC, the joints of the control system can arrive at the prescribed joint angular position at the predefined joint angular speed at the prescribed time.

The model of the robot can be found in Section 2. In Section 3, there is a designed fixed-time disturbance observer. The global terminal switching SMC (GTSSMC) based on EFDO is given in Section 4. The conclusion can be found in Section 5.

2. System model

The n-link robot manipulator is

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau + \tau_d \tag{1}$$

$q=\left(q_1,q_2,...,q_m\right)^T\in\mathbf{R}^m$ is the joint angular. $M(q)\in\mathbf{R}^{m\times m}$ which is the sum of $M_0(q)$ and $\Delta M(q)$ is the matrix, where $M_0(q)$ is the nominal part, $\Delta M(q)$ is the uncertain part. $C(q,\dot{q})\in\mathbf{R}^m$ which is the sum of $C_0(q,\dot{q})$ and $\Delta C(q,\dot{q})$ is the centripetal and coriolis forces, where $C_0(q,\dot{q})$ is the nominal part, $\Delta C(q,\dot{q})$ is the uncertain part. $G(q)\in\mathbf{R}^m$ which is the sum of $G_0(q)$ and $\Delta G(q)$ is the gravitational torque, where $G_0(q)$ is the nominal part, $\Delta G(q)$ is the uncertain part. $\tau\in\mathbf{R}^m$ is the control torque and $\tau_d\in\mathbf{R}^m$ is the disturbance with $\|\tau_d\|\le\overline{\tau}_d$. $F(q,\dot{q},\ddot{q})=-\Delta M(q)\ddot{q}-\Delta C(q,\dot{q})-\Delta G(q)\in\mathbf{R}^m$ is the uncertainty, $\|F(q,\dot{q},\ddot{q})\|\le b_0+b_1\|q\|+b_2\|\dot{q}\|^2=\overline{F}$ [4].

Let $q_r=\left(q_{1r},q_{2r},....,q_{mr}\right)^T\in\mathbf{R}^m$ which is the desired signal is twice differentiable signal, and define $e_1=q_r-q=\left(e_1^{(1)},e_2^{(1)},...,e_m^{(1)}\right)^T\in\mathbf{R}^m$, $e_2=\dot{q}_r-\dot{q}=\left(e_1^{(2)},e_2^{(2)},...,e_m^{(2)}\right)^T\in\mathbf{R}^m$, $\Gamma=M_0^{-1}(q)\left(C_0(q,\dot{q})+G_0(q)\right)$, then obtain

$$\begin{aligned}&\dot{e}_1=e_2\\&\dot{e}_2=\ddot{q}_r+\Gamma-M_0^{-1}(q)\tau-M_0^{-1}(q)\left(F+\tau_d\right)\end{aligned}\tag{2}$$

where $d=\left(d_1,d_2,...,d_m\right)^T=-M_0^{-1}(q)\left(F+\tau_d\right)\in\mathbf{R}^m$, $\|d\|\le\overline{d}=\left\|M_0^{-1}(q)\right\|\left(\overline{F}+\overline{\tau}_d\right)$ [1].

3. Entire fixed-time disturbance observer

The EFDO is designed as

$$\hat{d}=\left(d_1,d_2,...,d_m\right)^T\in\mathbf{R}^m\tag{3}$$

where

$$\begin{aligned}\hat{d}_k=&-\varepsilon_0^{(k)}\left(\mathrm{sig}^{\gamma_1^{(k)}}\left(\Lambda_k-e_k^{(2)}\right)+\mathrm{sig}^{\gamma_2^{(k)}}\left(\Lambda_k-e_k^{(2)}\right)\right)\\&-\left(\mu_0^{(k)}+\rho_0^{(k)}\right)\mathrm{sgn}\left(\Lambda_k-e_k^{(2)}\right)\\&-\left[\ddot{q}_r+\Gamma\right]_k\end{aligned}$$

$\varepsilon_0^{(k)}>0$, $\|d\|\le\mu_0^{(k)}$, $\rho_0^{(k)}=\left|\left[\ddot{q}_r+\Gamma\right]_k\right|$, $\dot{\Lambda}_k=\left[\ddot{q}_r+\Gamma\right]_k+\left[-M_0^{-1}(q)\tau\right]_k+\hat{d}_k$, $\gamma_1^{(k)}\in(0,1)$, $\gamma_2^{(k)}\in(1,+\infty)$, $k=1,2,...,m$.
The error of disturbance estimation is

$$\tilde{d}=\left(\tilde{d}_1,\tilde{d}_2,....,\tilde{d}_m\right)^T\in\mathbf{R}^m$$

where $\tilde{d}_k=\hat{d}_k-d_k=\dot{\Lambda}_k-\dot{e}_k^{(2)}:=\dot{s}_d^{(k)}$, $k=1,2,...,m$.

Theorem 1: For the system (2), the EFDO (3) accurately estimates uncertainties after a fixed time, despite the initial state.

Proof: If $V_d^{(k)}(t)\ge 0$ is a continuous function that satisfies

$$\begin{aligned}\dot{V}_d^{(k)}&=\mathsf{V}^{(k)}\left(V_d^{(k)}\right)\\&=\begin{cases}-2^{\frac{1+\gamma_1^{(k)}}{2}}\beta_d^{(k)}V_d^{(k)\frac{1+\gamma_1^{(k)}}{2}},V_d^{(k)}\in[0,0.5)\\-2^{\frac{1+\gamma_2^{(k)}}{2}}\beta_d^{(k)}V_d^{(k)\frac{1+\gamma_2^{(k)}}{2}},V_d^{(k)}\in[0.5,+\infty)\end{cases}\end{aligned}$$

where $\beta_d^{(k)}>0$, $k=1,2,...,m$, then $V_d^{(k)}$ converges to the equilibrium in finite time.

$$\tau_{ds}^{(k)}\left(V_{d0}^{(k)}\right)=$$
$$\begin{cases}\dfrac{1}{\beta_d^{(k)}\left(1-\gamma_1^{(k)}\right)}2^{\frac{1-\gamma_1^{(k)}}{2}}V_{d0}^{(k)\frac{1-\gamma_1^{(k)}}{2}}, V_{d0}^{(k)}\in[0,0.5)\\ \dfrac{1}{\beta_d^{(k)}\left(1-\gamma_2^{(k)}\right)}\left(2^{\frac{1-\gamma_2^{(k)}}{2}}V_{d0}^{(k)\frac{1-\gamma_2^{(k)}}{2}}-1\right)+\dfrac{1}{\beta_d^{(k)}\left(1-\gamma_1^{(k)}\right)}, V_{d0}^{(k)}\in[0.5,+\infty)\end{cases}$$

where $V_{d0}^{(k)}=V_d^{(k)}(0)$. Then,

$$\begin{aligned}\tau_{s\infty}^{(k)}&:=\lim_{V_{d0}^{(k)}\to+\infty}\tau_{ds}^{(k)}\left(V_{d0}^{(k)}\right)\\ &=-\frac{1}{\beta_d^{(k)}\left(1-\gamma_2^{(k)}\right)}+\frac{1}{\beta_d^{(k)}\left(1-\gamma_1^{(k)}\right)}\\ &\geq\tau_{ds}^{(k)}\end{aligned}$$

Consider the Lyapunov function $V'^{(k)}=\frac{1}{2}s_d^{(k)2}$. Let $\beta_d^{(k)}=\varepsilon_0^{(k)}$ and $V'^{(k)}(0)=V_0'^{(k)}$, and note that $\left|s_d^{(k)}\right|=2^{\frac{1}{2}}V'^{(k)\frac{1}{2}}$, then there is

$$\begin{aligned}\dot{V}'^{(k)}&=s_d^{(k)}\dot{s}_d^{(k)}\\ &=s_d^{(k)}\begin{pmatrix}-\varepsilon_0^{(k)}\left(\mathrm{sig}^{\gamma_1^{(k)}}\left(\Lambda_k-e_k^{(2)}\right)+\mathrm{sig}^{\gamma_2^{(k)}}\left(\Lambda_k-e_k^{(2)}\right)\right)\\ -\left(\mu_0^{(k)}+\rho_0^{(k)}\right)\mathrm{sgn}\left(\Lambda_k-e_k^{(2)}\right)\\ -[\ddot{q}_r+\Gamma]_k-d_k\end{pmatrix}\\ &=-\varepsilon_0^{(k)}\left(\left|s_d^{(k)}\right|^{\gamma_1^{(k)}+1}+\left|s_d^{(k)}\right|^{\gamma_2^{(k)}+1}\right)-\left(\mu_0^{(k)}+\rho_0^{(k)}\right)\left|s_d^{(k)}\right|\\ &\quad-[\ddot{q}_r+\Gamma]_k s_d^{(k)}-d_k s_d^{(k)}\\ &\leq-\varepsilon_0^{(k)}\left(\left|s_d^{(k)}\right|^{\gamma_1^{(k)}+1}+\left|s_d^{(k)}\right|^{\gamma_2^{(k)}+1}\right)\\ &=-2^{\frac{1+\gamma_1^{(k)}}{2}}\beta_d^{(k)}V'^{\frac{1+\gamma_1^{(k)}}{2}}-2^{\frac{1+\gamma_2^{(k)}}{2}}\beta_d^{(k)}V'^{\frac{1+\gamma_2^{(k)}}{2}}\\ &\leq V\left(V'\right)\end{aligned}$$

Therefore, the convergence time of $V'^{(k)}$

$$t_s\left(V_0'^{(k)}\right)\leq\tau_{ds}^{(k)}\left(V_0'^{(k)}\right)\leq\tau_{s\infty}^{(k)}$$

This completes the proof.

4. Global terminal switching sliding mode control based on EFDO

The EFDO-GTSSMC is designed as

$$u_G:u_{\sigma(t)} \tag{4}$$

where,

$$u_{\sigma(t)} := M_0(q)\left(\ddot{q}_r + \Gamma + \upsilon_{\sigma(t)}\right)$$

$$\upsilon_{\sigma(t)} := \left(\upsilon^{(1)}_{\sigma_1(t)}, \upsilon^{(2)}_{\sigma_2(t)}, ..., \upsilon^{(m)}_{\sigma_m(t)}\right)^T$$

$$\sigma_k(t) : [0, +\infty) \to I := \{1, 2\}$$

$$\upsilon^{(k)}_{i^*} := \frac{k^{(k)} q_k}{p_k}\left(\dot{e}^{(1)}_k - \frac{2K^{(k)}}{T_k} t + 2K^{(k)}\right)^{2-\frac{p_k}{q_k}} - \frac{2K^{(k)}}{T_k} + \hat{d}_k + \varepsilon_k \operatorname{sgn}\left(s^{(k)}_{i^*}\right)$$

$$\upsilon^{(k)}_{i_*} := \frac{k^{(k)} q_k}{p_k} \dot{e}^{(1)^{2-\frac{p_k}{q_k}}}_k + \hat{d}_k + \varepsilon_k \operatorname{sgn}\left(s^{(k)}_{i_*}\right)$$

$$s^{(k)}_{i^*} := k^{(k)}\left(e^{(1)}_k - \frac{K^{(k)}}{T_k} t^2 + 2K^{(k)} t - K^{(k)} T_k\right) + \left(\dot{e}^{(1)}_k - \frac{2K^{(k)}}{T_k} t + 2K^{(k)}\right)^{\frac{p_k}{q_k}}$$

$$s^{(k)}_{i_*} := k^{(k)} e^{(1)}_k + \dot{e}^{(1)\frac{p_k}{q_k}}_k$$

$k^{(k)} := h_k^{\frac{p_k}{q_k}}\left(e^{(1)}_k(0) - T_k K^{(k)}\right)^{\frac{p_k - q_k}{q_k}}, h_k := \dfrac{p_k}{t^{(k)}_p (p_k - q_k)}, K^{(k)} := \dfrac{-h_k e^{(1)}_k(0) - \dot{e}^{(1)}_k(0)}{2 - T_k h_k}$ with $2 - T_k h_k \neq 0$, $0 < T_k < t^{(k)}_p$, where $t^{(k)}_p$ is the prescribed system convergence time and T_k is the switching time, p_k and q_k are positive odd integers satisfying $1<p_k/q_k<2$, $\varepsilon_k>0$, $i^*=1$, $i_*=2$, $k=1,2,...,m$, and note that $\left.\left(-\dfrac{K^{(k)}}{T_k} t^2 + 2K^{(k)} t - K^{(k)} T_k\right)\right|_{t=T_k} = \left.\left(-\dfrac{2K^{(k)}}{T_k} t + 2K^{(k)}\right)\right|_{t=T_k} = 0$.

The EFDO-GTSS variable is defined as

$$s_G : s_{\sigma(t)} \tag{5}$$

where $s_{\sigma(t)} := \left(s^{(1)}_{\sigma_1(t)}, s^{(2)}_{\sigma_2(t)}, ..., s^{(m)}_{\sigma_m(t)}\right)^T$.

The EFDO-GTSSM is defined as

$$S_G : \quad s_{\sigma(t)} = 0 \tag{6}$$

The corresponding EFDO-GTSS manifold is

$$\mathsf{S}_G : \quad \left\{\left(e_1^T, e_2^T\right) \in \mathbf{R}^{2m} : s_{\sigma(t)} = 0\right\} \tag{7}$$

The scheduling strategy is

if $t \in T^{(k)}_i$ then $\sigma_k(t) = i$

where $T^{(k)}_1 := \{t : t \le T_k\}$, $T^{(k)}_2 := \{t : t > T_k\}$, i=1,2, and note that $s_G(0) = 0$.

Theorem 2: For the system (2), selecting the EFDO-GTSSMC (4), so that the control system has the global finite-time reaching characteristic, the property that the system convergence time can be prescribed, and the global robustness to uncertainties.

Proof: For the control system, there is

$$\begin{aligned}\ddot{e}_1 &= \dot{e}_2\\ &= \ddot{q}_r + \Gamma - M_0^{-1}(q)\tau - M_0^{-1}(q)\left(F+\tau_d\right)\\ &= \ddot{q}_r + \Gamma - M_0^{-1}(q)\left(F+\tau_d\right)\\ &\quad -M_0^{-1}(q)\left(M_0(q)\left(\ddot{q}_r+\Gamma+\upsilon_{\sigma(t)}\right)\right)\\ &= -\upsilon_{\sigma(t)} - M_0^{-1}(q)\left(F+\tau_d\right)\end{aligned}$$

$$\begin{aligned}\ddot{e}_k^{(1)} = &-\frac{k^{(k)}q_k}{p_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{2-\frac{p_k}{q_k}}+\frac{2K^{(k)}}{T_k}\\ &-\hat{d}_k-\varepsilon_k\,\mathrm{sgn}\left(s_{i^*}^{(k)}\right)-\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{aligned} \tag{8}$$

$$\begin{aligned}s_{i^*}^{(k)} &= k^{(k)}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)\\ &\quad+\frac{p_k}{q_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{\frac{p_k}{q_k}-1}\left(\ddot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}\right)\\ &= -\frac{p_k}{q_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{\frac{p_k}{q_k}-1}\left(\begin{array}{l}\hat{d}_k+\varepsilon_k\,\mathrm{sgn}\left(s_{i^*}^{(k)}\right)\\ +\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{array}\right)\end{aligned} \tag{9}$$

Consider the Lyapunov function

$$V_{i^*}^{(k)} = \frac{1}{2}s_{i^*}^{(k)2} \tag{10}$$

According to (9) and (10), there is

$$\begin{aligned}V_{i^*}^{(k)} &= s_{i^*}^{(k)}\dot{s}_{i^*}^{(k)}\\ &= s_{i^*}^{(k)}\left(-\frac{p_k}{q_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{\frac{p_k}{q_k}-1}\left(\begin{array}{l}\hat{d}_k+\varepsilon_k\,\mathrm{sgn}\left(s_{i^*}^{(k)}\right)\\ +\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{array}\right)\right)\\ &= -\frac{p_k}{q_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{\frac{p_k}{q_k}-1}\left(\begin{array}{l}\hat{d}_k s_{i^*}^{(k)}+\varepsilon_k\left|s_{i^*}^{(k)}\right|\\ +s_{i^*}^{(k)}\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{array}\right)\\ &\le -\frac{p_k}{q_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{\frac{p_k}{q_k}-1}\varepsilon_k\left|s_{i^*}^{(k)}\right|\\ &\le 0\end{aligned} \tag{11}$$

There is

$$\begin{aligned}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)' = &-\frac{k^{(k)}q_k}{p_k}\left(\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}\right)^{2-\frac{p_k}{q_k}}\\ &-\hat{d}_k-\varepsilon_k\,\mathrm{sgn}\left(s_{i^*}^{(k)}\right)-\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{aligned} \tag{12}$$

In the case of $\dot{e}_k^{(1)}-\frac{2K^{(k)}}{T_k}t+2K^{(k)}=0$ and $s_{i^*}^{(k)}\neq 0$, since

$$\left(\dot{e}_k^{(1)} - \frac{2K^{(k)}}{T_k}t + 2K^{(k)}\right)' = -\varepsilon_k \operatorname{sgn}\left(s_{i^*}^{(k)}\right) \neq 0 \tag{13}$$

$\dot{e}_k^{(1)} - \frac{2K^{(k)}}{T_k}t + 2K^{(k)}=0$ is not an attractor. Therefore, according to (11) and (12), the sliding manifold $s_{i^*}^{(k)} = 0$ is reached globally in finite time.

There is

$$\ddot{e}_k^{(1)} = -\frac{k^{(k)}q_k}{p_k}\dot{e}_k^{(1)^{2-\frac{p_k}{q_k}}} - \hat{d}_k - \varepsilon_k \operatorname{sgn}\left(s_{i^*}^{(k)}\right) - \left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k \tag{14}$$

$$\begin{aligned} s_{i_*}^{(k)} &= k^{(k)}\dot{e}_k^{(1)} + \frac{p_k}{q_k}\dot{e}_k^{(1)^{\frac{p_k}{q_k}-1}}\ddot{e}_k^{(1)} \\ &= -\frac{p_k}{q_k}\dot{e}_k^{(1)^{\frac{p_k}{q_k}-1}}\left(\hat{d}_k + \varepsilon_k \operatorname{sgn}\left(s_{i_*}^{(k)}\right) + \left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\right) \end{aligned} \tag{15}$$

Consider the Lyapunov function

$$V_{i_*}^{(k)} = \frac{1}{2}s_{i_*}^{(k)2} \tag{16}$$

According to (15) and (16), there is

$$\begin{aligned} V_{i_*}^{(k)} &= s_{i_*}^{(k)}\dot{s}_{i_*}^{(k)} \\ &= s_{i_*}^{(k)}\left(-\frac{p_k}{q_k}\dot{e}_k^{(1)^{\frac{p_k}{q_k}-1}}\begin{pmatrix}\hat{d}_k + \varepsilon_k \operatorname{sgn}\left(s_{i_*}^{(k)}\right) \\ +\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\end{pmatrix}\right) \\ &= -\frac{p_k}{q_k}\dot{e}_k^{(1)^{\frac{p_k}{q_k}-1}}\left(\hat{d}_k s_{i_*}^{(k)} + \varepsilon_k\left|s_{i_*}^{(k)}\right| + s_{i_*}^{(k)}\left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k\right) \\ &\leq -\frac{p_k}{q_k}\dot{e}_k^{(1)^{\frac{p_k}{q_k}-1}}\varepsilon_k\left|s_{i_*}^{(k)}\right| \\ &\leq 0 \end{aligned} \tag{17}$$

There is

$$\ddot{e}_k^{(1)} = -\frac{k^{(k)}q_k}{p_k}\dot{e}_k^{(1)^{2-\frac{p_k}{q_k}}} - \hat{d}_k - \varepsilon_k \operatorname{sgn}\left(s_{i_*}^{(k)}\right) - \left[M_0^{-1}(q)\left(F+\tau_d\right)\right]_k \tag{18}$$

In the case of $\dot{e}_k^{(1)}=0$ and $s_{i_*}^{(k)} \neq 0$, since

$$\ddot{e}_k^{(1)} = -\varepsilon_k \operatorname{sgn}\left(s_{i_*}^{(k)}\right) \neq 0 \tag{19}$$

$\dot{e}_k^{(1)}=0$ is not an attractor. Therefore, according to (17) and (18), the sliding manifold $s_{i_*}^{(k)} = 0$ is reached globally in finite time.

Since $s_G(0) = 0$, and according to (11), (12), (17), and (18), the EFDO-GTSS manifold can be reached globally in finite time, $s_G \equiv 0$, which means that the global robustness to uncertainties is provided, and the state will converge to the equilibrium at the prescribed convergence time. This completes the proof.

Theorem 3: Selecting the EFDO-GTSSMC (4) for the system (2), then the joints arrive at the prescribed joint angular position at the predefined joint angular speed at the prescribed time.

Proof: According to Theorem 2, there is $e_k^{(1)}=0$ and $\dot{e}_k^{(1)}=0$ for $t \geq t_p^{(k)}$. It means that the joints arrive at the prescribed joint angular position at the predefined joint angular speed at the prescribed time. This completes the proof.

5. Conclusion

In this paper, EFDO-GTSSMC for robot manipulators has been proposed. The control system has a global finite-time reaching characteristic, the property of invariance, and the system convergence time can be prescribed. The EFDO approximates uncertainties after a fixed time regardless of the initial state. The joint can arrive at the prescribed angular position at the prescribed time, with the predefined angular speed.

Disclosure statement

The author declares no conflict of interest.

References

[1] Zhang F, 2017, High-speed Nonsingular Terminal Switched Sliding Mode Control of Robot Manipulators. IEEE-CAA J. Autom Sinica, 4(4): 775–781.

[2] Deaecto GS, Souza M, Geromel JC, 2015, Discrete-time Switched Linear Systems State Feedback Design with Application to Networked Control. IEEE Trans. Automat. Control, 60(3): 877–881.

[3] Feng Y, Han FL, Yu XH, 2014, Chattering-free Full-order Sliding Mode Control. Automatica, 50(4): 1310–1314.

[4] Feng Y, Yu XH, Man ZH, 2002, Non-singular Terminal Sliding Mode Control of Rigid Manipulators. Automatica, 38(12): 2159–2167.

[5] Bartoszewicz A, 1996, Time-varying Sliding Modes for Second-order Systems. IEE Proceedings-Control Theory and Applications, 143(5): 455–462.

[6] Geng J, Wang YF, 2016, A Finite-time Sliding-mode Controller Design for Convergence at a Desired Time and Global Robust Control. Proceedings of the 35th Chinese Control Conference, Chengdu, China, 550–555.

[7] Hung JY, Gao WB, Hung JC, 1993, Variable Structure Control: A Survey. IEEE Trans. on Industrial Electronics, 40(1): 2–22.

[8] Vahidi-Moghaddam A, Rajaei A, Ayati M, et al., 2020, Adaptive Prescribed-time Disturbance Observer Using Nonsingular Terminal Sliding Mode Control: Extended Kalman Filter and Particle Swarm Optimization. IET Control Theory and Applications, 14(19): 3301–3311.

[9] Eskandari B, Yusefpour A, Ayati M, et al., 2020, Finite-time Disturbance-observer-based Integral Terminal Sliding Mode Controller for Three-phase Synchronous Rectifier. IEEE Access, 8: 152116–152130.