Journal of Architectural Research and Development, 2025, Volume 9, Issue 5
https://ojs.bbwpublisher.com/index.php/JARD
Online ISSN: 2208-3537
Print ISSN: 2208-3529

Prediction of Frozen Soil Deformation Characteristics Using Fractional Derivative Creep Model

Zhiheng Tian[1,2]*

[1]Future City Innovation Technology Co., Ltd., Shaanxi Construction Engineering Holding Group, Xi'an 710116, Shaanxi, China
[2]SCEGC-XJTU Joint Research Center for Future City Construction and Management Innovation, Xi'an Jiaotong University, Xi'an 710116, Shaanxi, China

Author to whom correspondence should be addressed.

Abstract: To investigate the temperature susceptibility and nonlinear memory effects of artificially frozen soil creep behavior, this study conducted uniaxial step-loading creep tests under controlled temperatures ranging from -10℃ to -20℃. The transient creep characteristics and steady-state creep rates of artificially frozen soils were systematically examined with respect to variations in temperature and stress. Experimental results demonstrate that decreasing temperatures lead to a decaying trend in the steady-state creep rate of silty frozen soil, confirming that low-temperature environments significantly inhibit plastic flow while enhancing material stiffness. Based on fractional calculus theory, a fractional derivative creep model was established. By incorporating temperature dependencies, the model was further improved to account for both stress and temperature effects. The model predictions align closely with experimental data, achieving over 91% agreement (standard deviation ± 1.8%), and effectively capture the stress-strain behavior of artificially frozen soil under varying thermal conditions. This research provides a reliable theoretical foundation for studying deformation characteristics in cold-regions engineering.

Keywords: Frozen soil; Fractional derivative; Creep deformation; Constitutive model

Online publication: October 10, 2025

1. Introduction

Driven by global climate change and the rapid development of cold-regions engineering, artificial ground freezing (AGF) technology has been extensively applied in metro connection passages, deep foundation pit support, and liquefied natural gas (LNG) storage tank foundations. As a critical supporting material in cold-regions underground engineering—including frozen construction methods and subsurface energy storage—the mechanical properties of artificially frozen soils, particularly their creep deformation behavior, directly govern the long-term stability and safety of structures. However, conventional integer-order creep models fail to accurately characterize the nonlinear deformation features and temperature susceptibility of frozen soils under low-temperature conditions, especially in capturing precise constitutive relationships under multiphysics field coupling.

Creep in artificially frozen soils exhibits marked nonlinearity, time-dependency, and temperature sensitivity. Its deformation mechanisms involve multiphysics coupling processes, including ice crystal redistribution, unfrozen water migration, and ice-soil skeleton interactions. Traditional integer-order models (e.g., Maxwell, Kelvin-Voigt, Burgers) can describe elastic and viscous responses but inadequately represent nonlinear memory effects and temperature-dependent characteristics during the attenuation creep stage. This theoretical limitation leads to insufficient prediction accuracy for long-term deformation in engineering practice, potentially causing safety hazards such as overdesigned freeze-wall thickness or support structure instability. Consequently, developing creep models that quantify thermo-stress coupling effects has become a priority in frozen soil mechanics.

Early frozen soil creep research primarily employed empirical and semi-empirical models. The Norton power-law model introduces a stress exponent to describe steady-state creep phase but fails to capture transient creep features [1]. While the Burgers model separates elastic, viscoelastic, and viscous deformation components, its linear superposition hypothesis deviates significantly from the nonlinear attenuation behavior characteristic of frozen soil creep. With advances in micromechanics and damage mechanics, scholars have attempted to enhance constitutive equations by incorporating damage variables. Fish's frozen viscoplastic model pioneered the inclusion of temperature as an independent variable [2]. However, its Arrhenius-type viscosity coefficient applies only to steady-state creep and does not account for strain hardening during transient stages.

Recently, fractional calculus has emerged as a powerful tool for describing non-integer-order dynamics due to its mathematical advantages in characterizing material heredity. Podlubny demonstrated that fractional derivatives inherently represent stress relaxation and creep history dependence through memory kernel functions, thereby providing new avenues for frozen soil creep modeling [3]. Wang *et al.* applied a fractional Scott-Blair model to analyze frozen clay creep, revealing a negative correlation between the fractional order α and unfrozen water content [4]. However, their model does not explicitly account for the influence of temperature gradients on α, limiting its engineering applicability.

To address these challenges in deformation regulation of artificially frozen soils under coupled thermal-mechanical fields, this study proposes a fractional derivative creep model capable of describing three-stage deformation processes (attenuation, steady-state, and accelerated creep) through a multiscale methodology integrating laboratory testing, numerical simulation, and field monitoring.

2. Uniaxial creep testing of artificially frozen soils

2.1. Experimental program

Soil specimens were retrieved from a cold-regions engineering site in Inner Mongolia, China, at depths of 15–20 m using core drilling equipment. The extracted frozen soil blocks were sealed in double-layer plastic bags and transported under refrigerated conditions (-30°C to -1°C) to the laboratory. After drying and pulverizing the undisturbed soil, specimens were reconstituted at 18.5% water content through compaction molding, forming cylindrical samples of 50 mm diameter × 100 mm height. These were subsequently flash-frozen at -30°C for 46 hours, followed by 24-hour isothermal curing at target test temperatures.

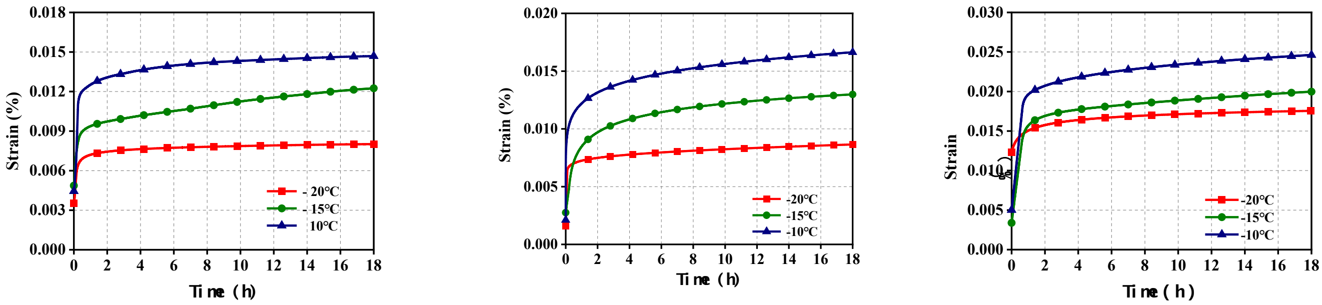
Uniaxial creep tests were conducted using a servo-hydraulic universal testing machine (UTM). Testing was performed at a baseline temperature of -10°C with extensions to -15°C and -20°C. A three-stage loading protocol was implemented by applying constant stresses at creep loading coefficients of $\sigma_c = 0.3\sigma_s$, $0.5\sigma_s$, $0.7\sigma_s$, where σ_s denotes the uniaxial compressive strength of frozen soil. Experimental data were acquired and strain-time curves were plotted, which enabled the quantitative analysis of how stress levels and test temperatures influence the creep behavior of frozen soils.

2.2. Experimental results

(1) Under constant stress conditions, the instantaneous strain of frozen soil increases significantly with rising temperature. For example, when the temperature increased from -20°C to -10°C, instantaneous strain rose by approximately 2.56 times (**Figure 1**). This results from elevated unfrozen water content, reduced viscosity at ice crystal interfaces, and weakened soil skeleton stiffness, collectively accelerating the initial elastic deformation response [5].

(2) At constant temperature (-20°C), creep deformation amplifies progressively with increasing stress levels, exhibiting intensified growth rates. When the loading coefficient reached 0.7, the maximum creep strain exceeded that at 0.3 by over 150%. This indicates significantly enhanced plastic flow in the ice matrix and shear-slip effects of unfrozen water films under high-stress conditions [6].

(3) Creep behavior under varying conditions demonstrates three distinct stages: (a) Attenuation creep stage: During initial loading, elastic deformation of the ice skeleton dominates. Creep rate decays exponentially with time, decreasing by ~40% per 5°C temperature increase. (b) Steady-state creep stage: Viscous flow of unfrozen water films and plastic slip of ice crystals reach dynamic equilibrium, stabilizing the creep rate. Steady-state rate correlates negatively with temperature and positively with stress (e.g., 30% higher at -10°C vs. -20°C). (c) Accelerated creep stage: Under high stress or elevated temperatures, microcrack propagation triggers chain damage mechanisms, causing creep rate to surge until failure. Here, the long-term strength limit and creep failure time obey a logarithmic correlation.



(a) $\sigma_c = 0.3\sigma_s$ (b) $\sigma_c = 0.5\sigma_s$ (c) $\sigma_c = 0.7\sigma_s$

Figure 1. Strain-time curves under constant stress conditions at varying temperatures

3. Parameter calibration for the fractional derivative creep model

3.1. Fractional derivative constitutive model for creep behavior

Under one-dimensional conditions, the creep strain of frozen soil can be expressed as:

$$\varepsilon_c = F(\sigma, T, t) \tag{1}$$

where ε_c represents the creep strain variable, and F denotes a function of stress (σ), temperature (T), and time (t).

To simplify the model, assuming decoupled effects of stress, temperature, and time, Equation (1) can be decomposed into:

$$\varepsilon_c = \sum_{i=1}^{n} f_i(T) g(\sigma) h(t) \tag{2}$$

Based on experimental results of frozen soil creep, the time-hardening theory was adopted to characterize creep behavior. This theory postulates that hardening is primarily governed by time and independent of accumulated deformation. Thus, the creep rate at constant temperature is expressed as:

$$\dot{\varepsilon}_{cr} = An\sigma^m t^{n-1} \tag{3}$$

where A is a material constant, n the stress exponent, and m the time decay exponent.

Assuming linear elastic instantaneous strain, integration of Equation (3) yields total strain:

$$\varepsilon_c = \varepsilon_{ct} + \varepsilon_{cr} = A + \int_0^t d\varepsilon_c = A + B\sigma_0^m t^n \tag{4}$$

where ε_c is the total strain, ε_{ct} the elastic strain, ε_{cr} the creep strain, and A, B, m, n are creep parameters.

It is evident that while this model accounts for load magnitude and creep effects, it fails to incorporate the influences of stress level and temperature level. Given that deformation in cold-regions engineering structures primarily arises from irreversible damage to the ice matrix under warming conditions, and considering the substantial temperature dependence of frozen soil creep behavior, we introduce stress level λ and temperature level θ. Consequently, an enhanced model incorporating these factors is formulated based on Equation (4), expressed as:

$$\varepsilon_c = \frac{\sigma}{E} + \lambda^k \times \theta \times \frac{t^{1-m}}{1-m} \tag{5}$$

$$\lambda = \frac{\sigma}{\sigma_c} \tag{6}$$

$$\sigma_c = \sigma_0 \times e^{(-\alpha(T-T_0))} \tag{7}$$

$$\theta = \frac{1}{1+\beta(T-T_{ref})} \tag{8}$$

where k is the stress coupling exponent, quantifying the nonlinear strengthening effect of stress level on creep rate; σ_c denotes the long-term strength of frozen soil; σ_0 represents the long-term strength at reference temperature T_0; α is the temperature sensitivity coefficient; β is the temperature decay coefficient; T_{ref} is the reference temperature.

3.2. Model parameter calibration

Long-term deformation of frozen soils is synergistically controlled by environmental temperature and stress levels, necessitating a fractional derivative creep model that concurrently captures stress thresholds and temperature sensitivity. In Equation (6) quantifying stress effects, σ_c represents the critical threshold for progressive damage within the ice matrix. Regarding temperature dependency in Equation (8), -10°C was selected as the reference temperature (T_0). To preserve the brittle-elastic characteristics of the ice skeleton, the instantaneous strain term is retained in the model. Parameter k governs ice crystal redistribution kinetics, while m characterizes the nonlinear influence of stress on unfrozen water film thickness.

Parameters α, β, k, and m were determined by curve-fitting experimental creep data under varying temperatures and stress levels. Values of α and β were calibrated through regression analysis of creep deformation against stress level λ and temperature level θ, as illustrated in **Figure 2**. Parameters k and m were obtained via multivariate least-squares fitting implemented in Python, with detailed results provided in **Table 1**.

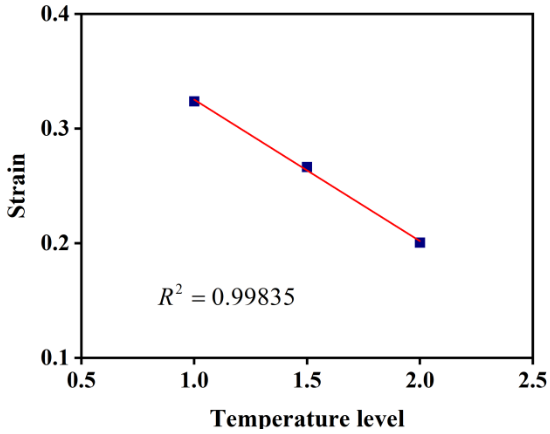


Figure 2. Relationship between creep strain and temperature level

Table 1. Model parameters fitting results

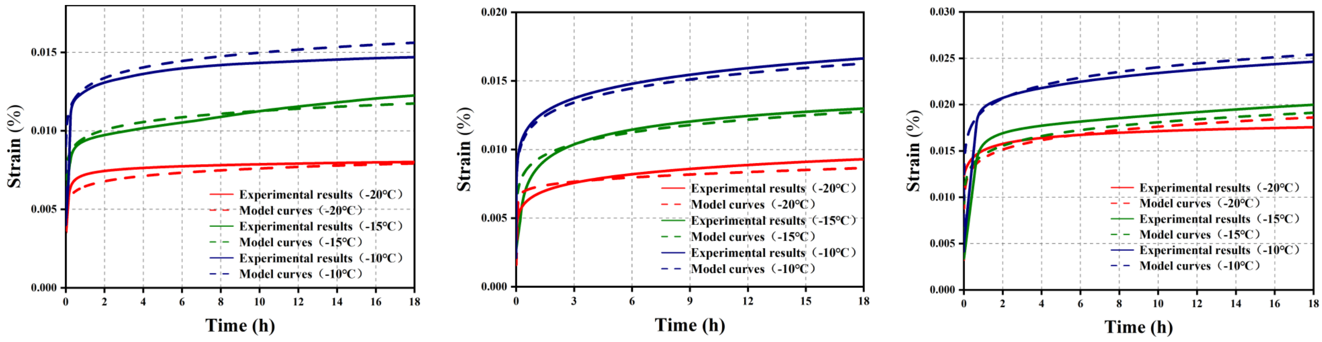
Temperature/℃	Loading coefficients	α	k	β	m
	0.3	0.55	0.05	1.62	0.21
-10	0.5	0.33	0.09	2.20	0.30
	0.7	0.17	0.11	2.25	0.12
	0.3	0.72	0.05	1.83	0.78
-15	0.5	0.42	0.09	2.14	0.46
	0.7	0.17	0.12	2.15	0.25
	0.3	0.72	0.08	1.82	0.78
-20	0.5	0.62	0.09	2.35	0.67
	0.7	0.27	0.05	2.15	0.41

3.3. Validation of fractional derivative creep model

Parameters from **Table 1** were substituted into Equation (5) to predict creep deformation of frozen soil under loading coefficients of 0.3, 0.5, and 0.7 using the fractional derivative creep model. As illustrated in **Figure 3**, close agreement exists between model predictions and experimental data, achieving an average goodness-of-fit of 91.9% (standard deviation: ±1.8%). This demonstrates the model's capability to effectively capture the creep behavior of frozen soils under varying thermo-mechanical conditions.

The model's high fitting accuracy stems from its mechanistic characterization of frozen soil creep deformation. Specifically, the temperature level θ couples temperature sensitivity, thereby quantifying the thickening effect of unfrozen water films [7]. The stress level λ captures the damage accumulation threshold, simulating accelerated propagation of ice crystal fractures to predict strain rate jumps. As λ increases, deformation rates during both attenuation and steady-state creep stages rise over time, causing the curve to approach an upward-sloping line. Crucially, λ governs the proportional durations of the three creep stages, manifesting as: (a) shortened steady-state creep duration, (b) earlier transition to accelerated creep, and consequently (c) extended acceleration periods.

Consequently, the fractional derivative creep model proposed herein accurately characterizes the full three-stage creep behavior—encompassing attenuation, steady-state, and accelerated creep—demonstrating broad applicability in permafrost engineering.



(a) $\sigma_c = 0.3\sigma_s$ (b) $\sigma_c = 0.5\sigma_s$ (c) $\sigma_c = 0.7\sigma_s$

Figure 3. Comparison between experimental results and model curves for frozen soil creep

4. Conclusion

To characterize creep deformation behavior of frozen soils, this study establishes an enhanced fractional derivative creep model incorporating coupled stress level λ and temperature reduction factor θ, based on uniaxial creep tests. Validated against experimental data through multivariate least-squares fitting, the model demonstrates both applicability and accuracy. The major findings are given as follows.

(1) The fractional derivative creep model achieves >91% average goodness-of-fit with experimental data. Consistent strain evolution patterns and robust performance across broad stress ranges and thermal conditions confirm its reliability for engineering deformation prediction.

(2) By introducing dual parameters λ (stress level) and θ (temperature reduction factor), the model quantifies stress-temperature coupling effects, accurately capturing stress-threshold behavior and temperature sensitivity. This enables precise characterization of full three-stage creep deformation.

(3) The model overcomes limitations of conventional empirical approaches where parameters lack physical significance due to pure curve-fitting. By explicitly incorporating thermo-mechanical controls, it exhibits enhanced generalizability for engineering applications.

Funding

National Key Research and Development Program of China "Structural Stability Assessment Techniques and Demonstration for Masonry Ancient Pagodas" (2023YFF0906005)

Disclosure statement

The author declares no conflict of interest.

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